Practice problems for Midterm 1

MAT 132

Oct 3, 2018

Name:	**ID #**:
(please print)	
Your recitation (e.g. R21):	(see list below)

This is a collection of practice problems for Midterm 1. Note that it doesn't mean that the actual midterm will contain exactly these kinds of problems or that it will be of the same length. The actual midterm will be shorter than this and may contain problems on any material covered in lectures; the best way to prepare for it is going over all homework assignments, not just over this collection of practice problems.

No notes, books, or calculators.

You must show your reasoning, not just the answer. Answers without justification will get only partial credit. Your solutions should be written so that the grader is able to follow your reasoning and computations.

Please cross out anything that is not part of your solution — e.g., some preliminary computations that you didn't need. **Everything not crossed out will be considered part of your solution and graded. If the grader can't understand or follow it, points will be taken off.**

When computing numerical answers, please simplify the answers as much as possible (e.g., $\sin(0)$ should be replaced by 0). However, do not replace algebraic expressions and constants such as $\sqrt{2}$ or π by approximate values.

When computing indefinite integrals, do not forget the integration constant!

1. Calculate the following integrals:

(a)
$$\int (e^{3x+1} + \sin^2 x)\, dx$$

(b)
$$\int x^2 \sin x\, dx$$

(c)
$$\int \tan x\, dx$$

(d)
$$\int \frac{dx}{x^2 + 5x + 6}$$

2. Determine whether the following improper integrals are convergent or divergent and evaluate those that are convergent:

(a)
$$\int_{-1}^{1} \frac{dx}{2x - 1}$$

(b)
$$\int_{1}^{\infty} \frac{\sqrt[3]{\ln x}}{x}\, dx$$

(c)
$$\int_{-\infty}^{\infty} \frac{x^2}{1 + x^6}\, dx$$

(d)
$$\int_{0}^{\pi/2} \tan x\, dx$$

3. Find numbers a and b such that
$$a \leq \int_{\pi/6}^{\pi/2} \frac{5}{\sin x}\, dx \leq b.$$

4. Find the area of the region enclosed by the curves $y = x^3$ and $y = \sqrt{x}$ in two different ways, by integrating in x and by integrating in y.

5. Find the volume of the solid obtained by rotating the finite region bounded by the curves $y = \tan x$, $y = 0$, $x = \frac{\pi}{4}$ about:

(a) the x-axis
(b) the line $y = -1$

6. Calculate the following definite integrals:

(a)
$$\int_0^2 \sqrt{4-x^2}\,dx$$

(b)
$$\int_0^1 \ln x\,dx$$

(c)
$$\int_0^\pi (\sin x)^3\,dx$$

7. Find the length of a curve given parametrically by $x(t) = e^t \sin t$, $y(t) = e^t \cos t$, $0 \le t \le \pi$.