Problem 7 Compute the derivative of the function

$$\ln(\tan^2(x))$$

and simplify your answer.

$$\left(\ln(\tan^2(x))\right)'$$

$$= \frac{1}{\tan^2(x)} \cdot (\tan^2 x)'$$

$$= \frac{2\tan(x)}{\tan^2(x)} \cdot (\tan x)'$$

$$= \frac{2}{\tan(x)} \sec^2 x$$

$$= \frac{2}{\frac{\sin x}{\cos x} \cdot \cos^2 x} = \frac{2}{\sin x \cos x} \left(= \frac{4}{\sin 2x}\right)$$

$$\left(= 4\csc 2x\right)$$

Problem 8 Estimate the integral

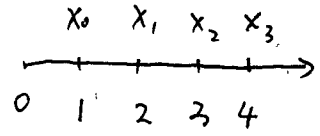
$$\int_1^4 x \ln x dx$$

using three rectangles and

a) right endpoints b) left endpoints.

c) Are your answers in a) and b) over- or under-estimates of the actual integral?

(Hint: you may want to determine whether the function $f(x) = x \ln x$ is increasing or decreasing)



$\Delta x = 1$

a) $R_3 = \sum_{i=1}^{3} x_i \ln x_i = 2\ln 2 + 3\ln 3 + 4\ln 4$

$= 10 \ln 2 + 3\ln 3$

b) $L_3 = \sum_{i=0}^{2} x_i \ln x_i = 1\ln 1 + 2\ln 2 + 3\ln 3$

$= 2\ln 2 + 3\ln 3$

c) Since $(x\ln x)' = \ln x + 1 > 0$ on $[1,4]$,

it's increasing.

So. R_3 is overestimate.

L_3 is underestimate.