

# Practice Final Exam MAT 125

May 8, 2006

Name: \_\_\_\_\_ ID number: \_\_\_\_\_

Recitation number (e.g., R01): \_\_\_\_\_

(for evening lecture, use “ELC 4”)

<b>Lecture 1</b>	MWF 9:35–10:30	An, Daniel
R01	M 11:45am–12:40pm	Solorzano, Pedro
R02	Th 3:50pm– 4:45pm	Ostrovsky, Stanislav
R03	W 11:45am–12:40pm	Solorzano, Pedro
R04	Tu 11:20am–12:15pm	Basu, Somnath
R05	Tu 11:20am–12:15pm	Han, Zhigang
R31	M 10:40am–11:35am	Patu, Ionel
<b>Lecture 2</b>	TuTh 2:20pm – 3:40pm	Kirillov, Alexander
R06	M 11:45am–12:40pm	Zeng, Huayi
R07	F 11:45am–12:40pm	Nowicki, Jan
R08	W 9:35am–10:30am	Ma, Xin
R09	Tu 3:50pm– 4:45pm	Ostrovsky, Stanislav
R10	F 8:30am–9:25am	Ma, Xin
<b>Lecture 3</b>	MW 3:50pm–5:10pm	Chen, Je-Wei
R11	M 9:35am–10:30am	Poole, Thomas
R12	F 10:40am–11:35am	Panok, Lena
R13	W 2:20pm–3:15pm	Poole, Thomas
R14	Tu 11:20am–12:15pm	Lyberg, Ivar
R15	Th 11:20am–12:15pm	Lyberg, Ivar
R32	M 2:20pm– 3:15pm	Guo, Weixin
<b>Evening Lec 4</b>	TuTh 6:50pm–8:10pm	Bulawa, Andrew

Please answer each question in the space provided. Please write full **solutions**, not just answers. Unless otherwise marked, **answers without justification will get little or no partial credit**. Cross out anything the grader should ignore and circle or box the final answer. Do **NOT** round answers.

No books, notes, or calculators!

**Do not open the exam until instructed by proctor!**

1. Compute the following limits. Please distinguish between “ $\lim f(x) = \infty$ ”, “ $\lim f(x) = -\infty$ ” and “limit does not exist even allowing for infinite values”.

(a)  $\lim_{x \rightarrow -1} x^2 + x - 1$

(b)  $\lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{x + 3}$

(c)  $\lim_{t \rightarrow 0} \frac{\sqrt{2-t} - \sqrt{2}}{t}$

(d)  $\lim_{x \rightarrow 0} x \sin \pi \left( x^2 + \frac{1}{x^2} \right)$

(e)  $\lim_{x \rightarrow \infty} \frac{x^3 + 2x + 1}{x^3 - 2x + 1}$

(f)  $\lim_{x \rightarrow \pi/2} \frac{\cos x}{2x - \pi}$

2. Compute the derivatives of the following functions

(a)  $f(x) = x^3 - 12x^2 + x + 2\pi$

(b)  $f(x) = (2x + 1) \sin(x)$

(c)  $g(s) = \sqrt{1 + e^{2s}}$

(d)  $h(t) = \frac{1+e^t}{1-e^t}$

(e)  $f(x) = (2x + 2)^{10}$

(f)  $g(x) = x^{(\sin x)}$

3. Let  $f(x) = xe^{-x^2}$ .

(a) Find asymptotes of  $f(x)$  (hint:  $f(x) = \frac{x}{e^{(x^2)}}$ )

(b) Compute the derivative of  $f(x)$

(c) On which intervals is  $f(x)$  increasing? decreasing?

(d) Sketch a graph of  $f(x)$  using the results of the previous parts and the fact that  $f(0) = 0$ .

4. Let  $f(x) = \frac{1}{\sqrt{1+x}}$ . Write the linear approximation for  $f(x)$  near  $x = 0$  and use it to estimate  $f(0.1)$ .

5. Let  $f(x) = -2x^3 + 6x^2 - 3$ .

(a) Compute  $f'$ ,  $f''$ .

(b) On which intervals is  $f(x)$  increasing/decreasing?

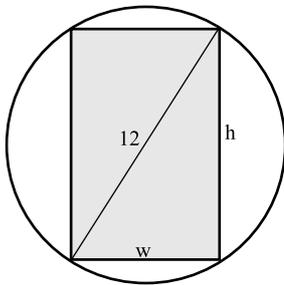
(c) On which intervals is  $f(x)$  concave up/down?

(d) Find all critical points of  $f(x)$ . Which of them are local maximums? local minimums? neither? Justify your answer.

6. It is known that the polynomial  $f(x) = x^3 - x - 1$  has a unique real root. Between which two whole numbers does this root lie? Justify your answer.

7. It is known that for a rectangular beam of fixed length, its strength is proportional to  $w \cdot h^2$ , where  $w$  is the width and  $h$  is the height of the beam's cross-section.

Find the dimensions of the strongest beam that can be cut from a 12" diameter log (thus, the cross-section must be a rectangle with diagonal 12").



8. The curve defined by the equation

$$y^2(y^2 - 4) = x^2(x^2 - 5)$$

is known as the "devil's curve". Use implicit differentiation to find the equation of the tangent line to the curve at the point  $(0; -2)$ .