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Calculus I
MAT 131
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Practice Midterm 2

1. Compute derivatives of the following functions:

$$y = \frac{x^3 - 3x^2 + 3x - 1}{x - 1}, \quad y = \sin x^2 - \sin^2 x, \quad y = \sqrt{x}^{1/x}, \quad y = \cos(\arctan x), \quad y = \arctan(\cos x),$$

$$y = \sin e^x, \quad y = e^{\sin x}, \quad y = \ln\sqrt{x^2+1}, \quad y = 3^{4^x}, \quad y = \log_5(\log_2 x), \quad y = \frac{x^2 e^x}{x^2 + 2x + 1},$$

$$y = \arcsin\left(\frac{x}{\pi}\right), \quad y = \sqrt{\tan x^3}, \quad y = \sqrt{\tan^3 x}, \quad y = (\cos x)^{\sin x}$$

2. Find the second derivatives of the following functions:

$$y = xe^{x^2}, \quad y = (1 + x^3)^{-1}, \quad y = (1 + x^2)\arctan x.$$

3. Calculate $\dfrac{d^2y}{dx^2}$ if $e^{x+y} = xy$.

4. Prove that $\sqrt[n]{a^n + x} \approx a + \dfrac{x}{na^{n-1}}$ if $a > 0$ and x is near 0.

5. Use linearization to calculate an approximate value of $\arctan 1.05$. Is your estimate an underestimate or an overestimate?

6. Show that the curve $y^3 - x^2 \sin y = -1$ has a horizontal tangent line at point $x = 0$, $y = -1$.

7. Find the equation of the tangent line to the parametric curve $x(t) = \dfrac{1}{t+1}$, $y(t) = \dfrac{1}{t-1}$ at the point $x = 1/3$, $y = 1$.

8. Find the equation of the tangent line to the curve $x^2 = y(y-1)(y+3)$ at the point $x = 6$, $y = 3$.

9. A particle moves to the right along the line $y = 3$ meters. At the moment when the distance from the origin to the particle equals 5 meters, this distance is increasing at the rate of 8 meters per second. Find the speed of the particle at this moment.

10. A good collection of related rates problems is in the textbook. Take, for example, these: Sec. 4.1 no. 1-4, 9-13.