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Calculus I
MAT 131
February 9, 2009

Practice Midterm 1

1. Draw the graphs of the following functions:

a) $y = x^2 - 3x + 2$ **b)** $y = |-x^2 - x + 6|$ **c)** $y = \left(\frac{1}{2}\right)^{|x|}$ **d)** $y = |\ln x|$ **e)** $y = \ln|x|$

f) $y = \ln|x+1|$ **g)** $y = \ln\frac{1}{x}$

2. Find the inverse of the following functions or explain why the inverse does not exist. Draw graphs of a function and its inverse on the same picture.

a) $y = \frac{1}{x+2},\ x > -2$ **b)** $y = x^2 + 4x + 5$ **c)** $y = x^2 + 4x + 5,\ x \geq -2$ **d)** $y = |x-1|$

3. Let $f(x) = 2x + 1, \quad g(x) = x^2$. Find $f^{-1} \circ g$.

4. Calculate the following limits or explain why they do not exist:

a) $\lim\limits_{x\to 0} \frac{|x^2-1|}{x+1}$ **b)** $\lim\limits_{x\to 1} \frac{|x^2-1|}{x+1}$ **c)** $\lim\limits_{x\to -1} \frac{|x^2-1|}{x+1}$ **d)** $\lim\limits_{x\to \infty} \frac{|x^2-1|}{x+1}$ **e)** $\lim\limits_{x\to -\infty} \frac{|x^2-1|}{x+1}$

f) $\lim\limits_{x\to\infty} \left(\frac{x^3}{x^2+1} - x\right)$ **g)** $\lim\limits_{x\to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$ **h)** $\lim\limits_{x\to e} \frac{\cos x}{\ln x}$ **i)** $\lim\limits_{x\to 0^+} \frac{\cos x}{\ln x}$ **j)** $\lim\limits_{x\to 1} \frac{\cos x}{\ln x}$

5. For a given function, find its domain, zeros, discontinuities and their types, equations of all vertical, horizontal and oblique asymptotes. Draw the graph.

a) $f(x) = \frac{x^2+x}{x^2-4x+3}$ **b)** $f(x) = \frac{x^3+3x^2+2x}{x^2-4x+3}$ **b)** $f(x) = \frac{5x^3+2x}{x^3-x}$.

6. Sketch the graph of a function f possessing the following properties:
1) f is even
2) f is continuous everywhere except two points
3) $\lim\limits_{x\to -2^+} f = -\infty$, $\lim\limits_{x\to -2^-} f = +\infty$
4) an oblique asymptote for the graph of f at $+\infty$ is $y = x + 1$
5)$f(0) = 1$
6)$f(3) = 4$.

7. For which value of a constant a is the function

$$f(x) = \begin{cases} x^3 + 1, & x \geq -1 \\ ax^2 - 1, & x < -1 \end{cases}$$

continuous? Draw the graph of f for such a.

8. Show that the equation $x^3 + 3x^2 + 4x - 5 = 0$ has a root in the interval $(0, 1)$.

9. Let $f(x) = \dfrac{1}{x+1}$. Find the difference quotient for $f(x)$ at point $x = -2$. Find the derivative of $f(x)$ at $x = -2$ as the limit of the difference quotient. Explain a geometrical meaning of $f'(-2)$ (draw a picture).

10. Let

$$f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

Show that f is continuous at any x. Is f differentiable at $x = 1$? If so, find $f'(1)$ using the definition of derivative.

11. Let $y = x^3 - x$. For $x = 2$ and $\Delta x = 1$, calculate Δy and dy. Do the same for $x = 2$ and $\Delta x = 0.1$. Draw pictures for both cases.