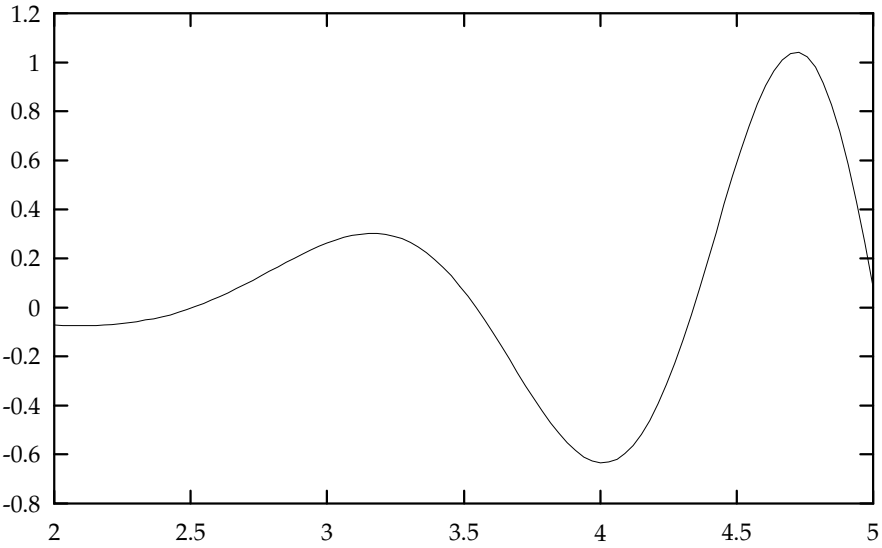
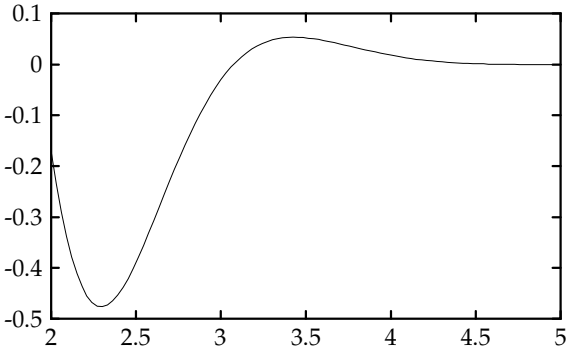
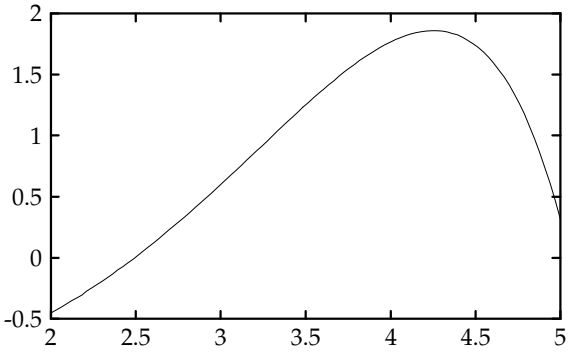
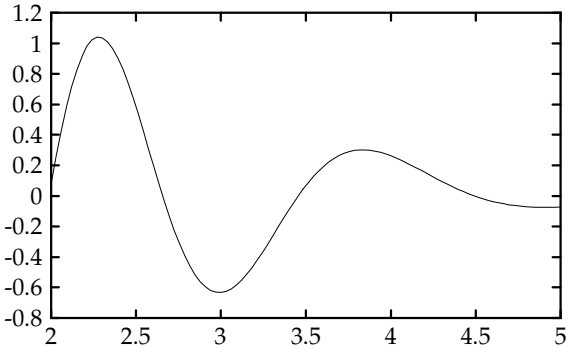
Problem 1. Below is a sketch of the graph of a function f.

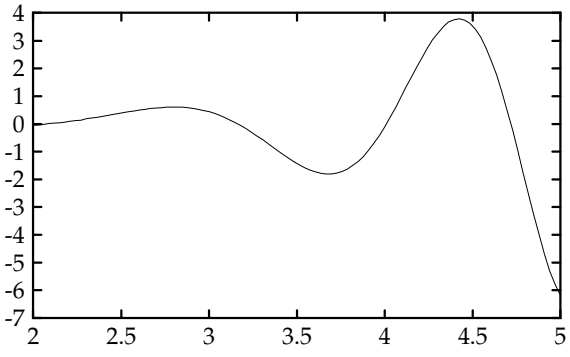


Which is the graph of f'?









Problem 2. Let f and g be two functions satisfying $f(2) = 5$, $f'(2) = 2$, $g(2) = -3$, and $g'(2) = 4$. Find

(a) $\lim_{h \to 0} \dfrac{g(2+h) - g(2)}{h}$

(b) $\lim_{x \to 2} f(x)$

(c) $\left(\dfrac{f}{g}\right)'(2)$

(d) $\lim_{h \to 0} \dfrac{f(2+h)g(2+h) + 15}{h}$

Problem 3. Compute

(a) $g'(x)$ if $g(x) = \dfrac{x^5 \cos(x)}{1 + x + x^8}$.

(b) $\dfrac{d^2y}{dx^2}$ if $y = e^{\sin(x)}$.

(c) $\lim\limits_{x \to 0} \dfrac{\ln(1+x)}{x}$.

(d) $\lim\limits_{x \to 0} \dfrac{\cos(x) - 1}{\sin(x)}$.

Problem 4. Suppose that air is being pumped into a spherical ballon at a rate of 20cm^3 per second.

(a) How fast is the radius growing when the volume is $1000\pi\text{cm}^3$?

(b) How fast is the surface area growing at this time?

Problem 5. Consider a function A with the following properties:

- $A(1) = \dfrac{\pi}{4}$,
- $\lim\limits_{x\to\infty} A(x) = \dfrac{\pi}{2}$ and $\lim\limits_{x\to-\infty} A(x) = -\dfrac{\pi}{2}$
- $A'(x) = \dfrac{1}{1+x^2}$ for all $x \in (-\infty, \infty)$.

(a) There isn't a nice formula for $A(x)$ so, it is not possible to determine $A(1.2)$ exactly. Use an approximation by differentials to estimate $A(1.2)$.

(b) On what interval is the graph of A concave down?

(c) Multiple choice: circle the correct one. $\dfrac{d}{dx}\left(A\left(2\dfrac{\sin(x)}{\cos(x)}\right)\right) =$

$$\frac{2}{1+3\sin^2(x)} \qquad \frac{2\sec(x)\tan(x)}{1+x^2} \qquad 4x^2 \qquad \frac{1}{1+4\tan^2(x)}$$

EXAM

Sample Midterm 2

Math 131

November 3, 2003