Problem 1. Let $r(x) = \dfrac{2x}{x^2+1}$.

(a) Use the definition of the derivative to compute $r'(a)$.

(b) There are two points at which the tangent line to the graph of r is horizontal. Give the coordinates of these points.

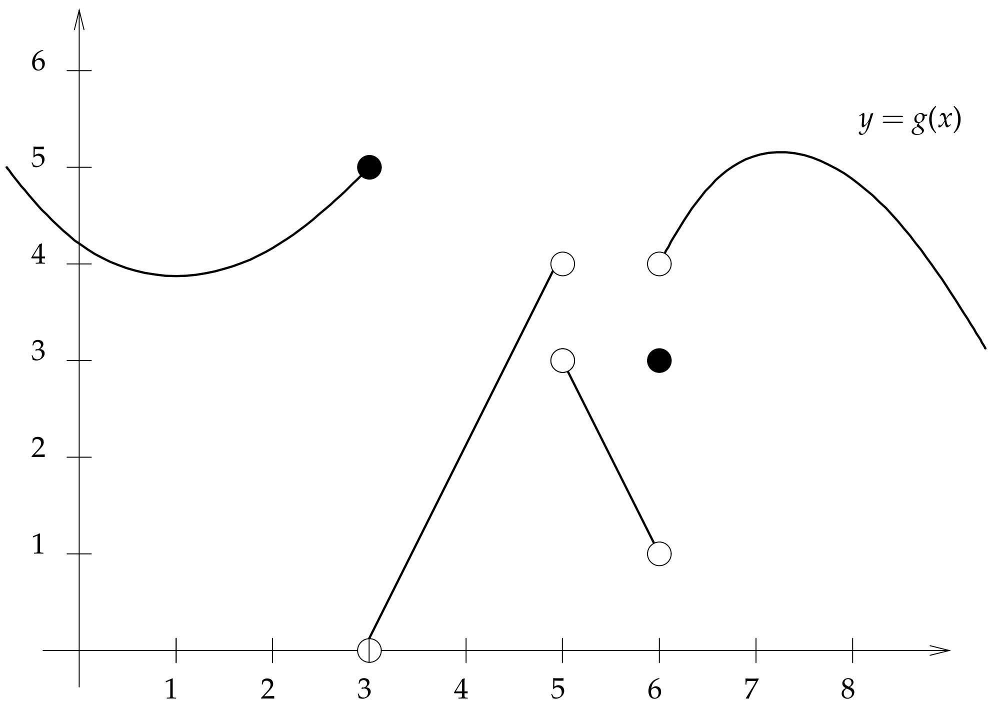
Problem 2. Compute:

(a) $\lim_{x\to\infty} \frac{1}{x^2}\sin(x)$

(b) $\lim_{t\to 4} \frac{\sqrt{t+5}-3}{\sqrt{2t+1}-3}$

(c) $\lim_{x\to 2^-} \frac{x^2-3x}{x^2-4}$

Problem 3.



Use the picture to find:

(a) $\lim_{x \to 6^+} g(x)$

(b) $g(6)$

(c) $\lim_{x \to 6} g(x)$

(d) $g(5)$

(e) $\lim_{x \to 3^-} g(x)$

(f) $\lim_{x \to 3^+} \frac{1}{g(x)}$

(g) $\lim_{x \to 3^+} \frac{x-3}{g(x)}$

(h) $\lim_{h \to 0} \frac{g(4+h)-2}{h}$

(i) $\lim_{x \to 3^-} g(2x)$

Problem 4. A recent study of toxin levels in a stream has yielded the following data:

t	1	2	2.1	4	6.5	7.2	8	9	10	10.9	11
$a(t)$	3.12	5.4	5.7	8.15	10.10	9.63	9.21	9.31	8.98	7.82	7.62

Here, $a(t)$ is the amount of toxin measured at time t. Use the table to estimate $a'(2)$ and $a'(11)$.

Problem 4. True or False

(a) If f is any function with $f(3) = 5$ and $f(6) = 3$, then there must exist a number x between 3 and 6 with $f(x) = 4$.

(b) The function f defined by

$$f(x) = \begin{cases} x^2 + 2x + 1 & \text{if } x \leq 1, \\ 5x - 1 & \text{if } x > 1 \end{cases}$$

is continuous at $x = 1$.

(c) $\lim_{r \to 0} \frac{1}{r^2} = \infty$.

(d) If $|x - 3| < 0.1$ then $9.9 < 3x + 1 < 10.1$.

(e) If $\lim_{x \to -\infty} g(x) = 14$ then the line $y = 14$ is a horizontal asymptote of the graph of g.

(f) The curve $y = \frac{x^3 - 1}{x^2 - 1}$ has a vertical asymptote at $x = 1$.

(g) The $\lim_{t \to 0} \frac{e^t - 1}{t - 1}$ exists and is finite.

EXAM

Sample Midterm 1

Math 131

October 3, 2003