Problem 1. Evaluate. Justify your answers.

(a) $\lim_{x \to 2} \dfrac{x^4 - 16}{x^2 - 4} =$

(b) $\lim_{x \to 6} \dfrac{x - 6}{\sqrt{2x - 3}} =$

(c) $\lim_{t \to \infty} \dfrac{3t^3 + t - 5}{4t^3 + t^2 + 6} =$

(d) $\lim_{x \to \infty} \dfrac{x^2 - x}{x + 10\sqrt(x)} =$

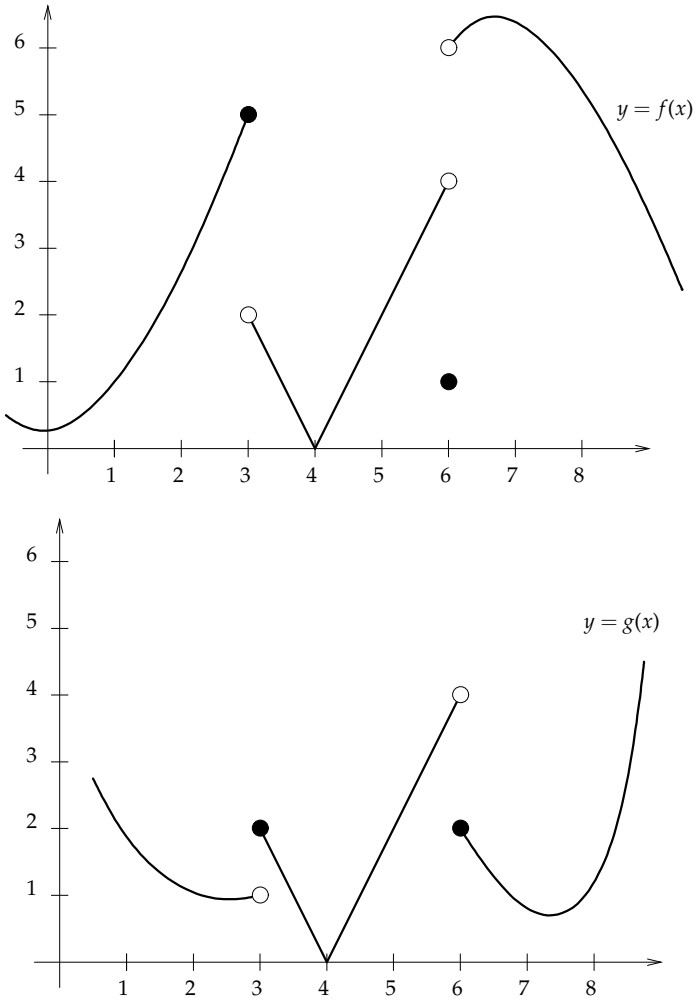
Problem 2. The function f defined by $f(x) = 3x - 1$ is continuous at $x = 2$. Intuitively, this means that $f(x)$ will be near $f(2) = 5$ whenever x is near 2. More precisely, we have

$$\lim_{x \to 2} f(x) = 5 \text{ and } f(2) = 5.$$

Your problem: find a number δ so that

$$\text{if } 2 - \delta < x < 2 + \delta \text{ then } 4.8 < f(x) < 5.2.$$

Problem 3.



Use the picture to find:

(a) $\lim_{x \to 3^+} g(x) =$

(b) $f(3) =$

(c) $\lim_{x \to 3} f(x) =$

(d) $\lim_{x \to 6} (f+g)(x) =$

(e) $(f+g)(6) =$

(f) $\lim_{x \to 4} \dfrac{f(x)}{g(x)} =$

Hint: The answers are (out of order) 1, 2, 3, 5, 8, and "does not exist."

Problem 4. True or False. No justification is required.

(a) If f is a continuous function and $\lim_{x\to 1} f(x) = 5$ then $f(1) = 5$.

(b) Let f be continuous on $[a, b]$. Then, the intermediate value theorem says that for any $c \in (a, b)$, $f(c)$ is necessarily between $f(a)$ and $f(b)$.

(c) $\lim_{t\to 0} t \sin\left(\frac{1}{t}\right) = 0$.

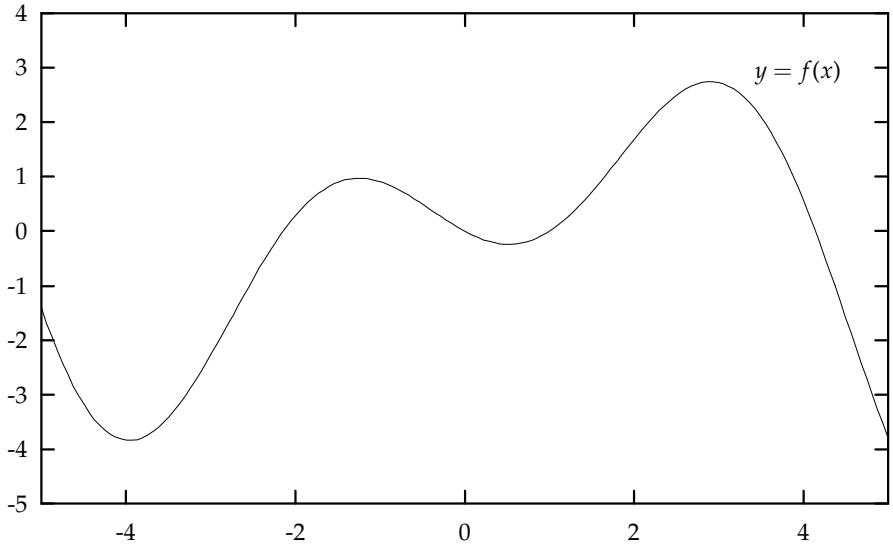
(d) The equation $x^5 + x - 2 = 2$ has a solution in the interval $(0, 3)$.

(e) $\lim_{x\to 0} \frac{1}{x} = +\infty$.

(f) Let g be defined by $g(x) = \sqrt[5]{8\pi}$. Then the tangent line to the graph of g is horizontal at every point.

Problem 5. Let $f(x) = \dfrac{3}{x+1}$. Use the definition of the derivative to compute $f'(1)$. Show your work.

Problem 6.



Use the sketch of $y = f(x)$ above to order the following numbers from smallest to largest:

$$f'(-4),\ f'(-2),\ f'(0),\ f'(4).$$

EXAM

Midterm 1

Math 131

October 9, 2003

- Name .....
- Student ID .....
- Lecture Section .....
- Recitation Section .....

- Neatness counts.
- Each part of each problem is worth 5 points, except for problem 5, which is worth 10 points. The total possible score is 100.
- You have 90 minutes to complete this exam.