Problem 1.

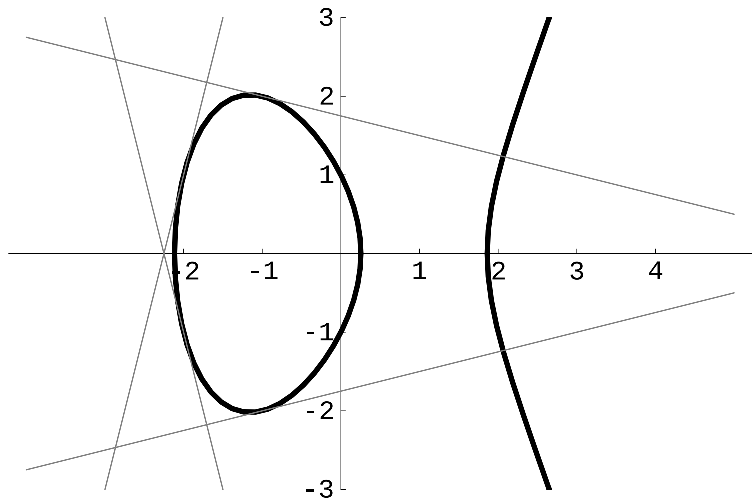
(a) Use the definition of the derivative to compute $h'(x)$ if $h(x) = \dfrac{1}{x^2}$.

(b) Use the definition of the integral to compute $\displaystyle\int_{-1}^{3} (2x+1)\, dx$.

Problem 2. Below is a picture of the *elliptic curve* defined by the equation

$$y^2 - 1 = x^3 - 4x$$

and four lines tangent to the curve. Pick two of these lines and give their equations. Clearly indicate which of your equations describe which lines.

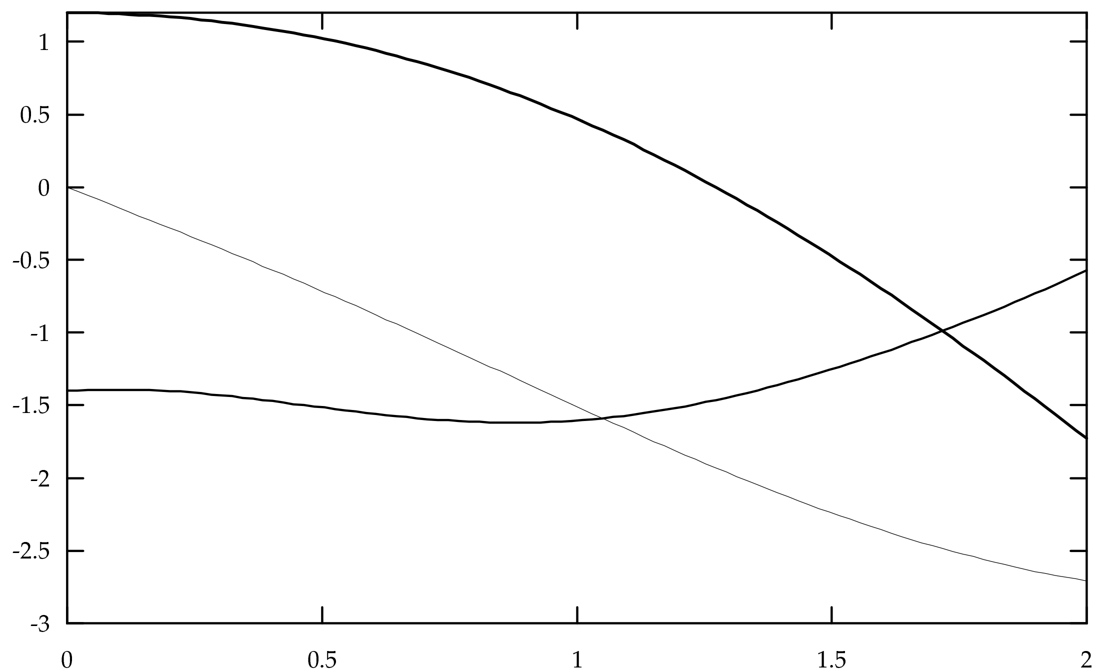


Problem 3. Let f be defined by

$$f(x) = \frac{x^2 - x - 2}{3x^2 - 4x - 4}.$$

Give the equations of all horizontal and vertical asymptotes of the graph of f.

Problem 4. Below, portions of the graphs of f, f', and f'' are sketched. Which are which? Give brief, but conclusive, evidence supporting your answer.



Problem 5. Match:

(a) $\lim_{h\to 0} \frac{\sin(2xh+h^2)}{h}$

(b) $\lim_{h\to 0} \frac{1}{h}\int_x^{x+h} \sin^3(t)\,dt$

(c) $D_x\left(\tan^2(x)-\sec^2(x)\right)$

(d) $D_x\, f(\sqrt{x}))$

(e) $\int_{\frac{1}{8}}^{\frac{1}{3}} \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx$

(f) $f'\left(\frac{\pi}{2}\right)$ if $f(x)=\left(\frac{x}{\pi}\right)^3\cos(x)-4x$

(g) $D_x\int_2^4 2t\,dt$

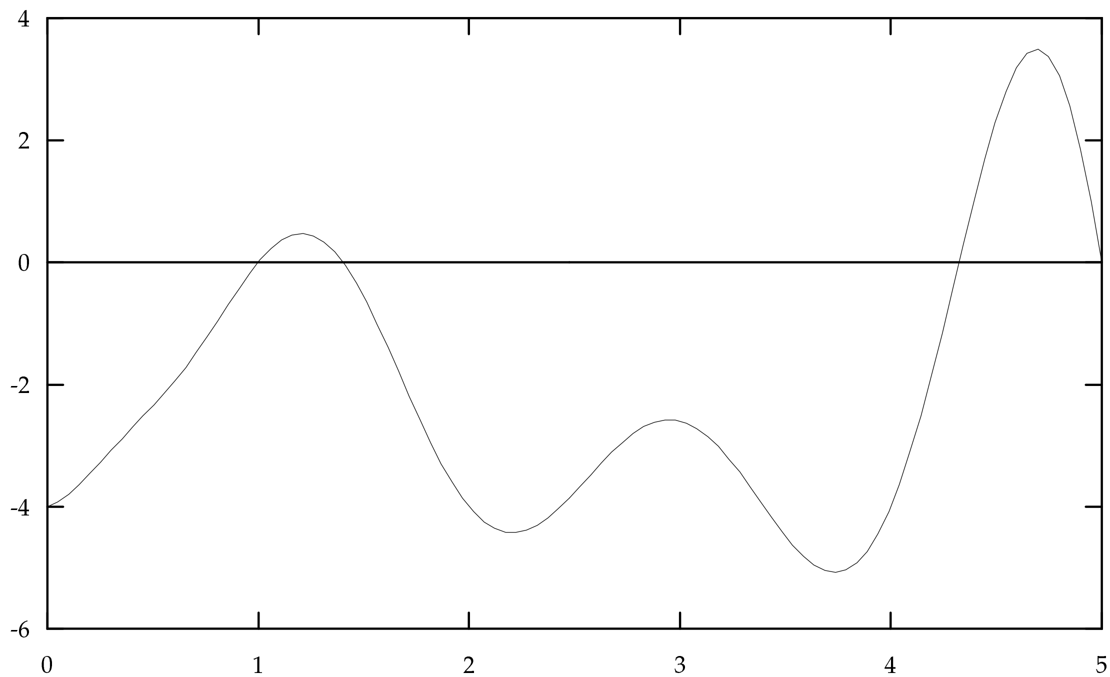
(h) $\int \left(x^3\cos(x)+3x^2\sin(x)\right)dx$

(i) $D_x\sqrt{f(x)}$

The answers (out of order, of course) are:

$$0,\quad 0,\quad 2x,\quad \frac{38}{3},\quad -\frac{33}{8},\quad x^3\sin(x),\quad \sin^3(x),\quad \frac{f'(x)}{2\sqrt{f(x)}},\quad \frac{f'(\sqrt{x})}{2\sqrt{x}}.$$

Problem 6. Below is the graph of a function f.



(a) Is $\int_0^5 f(x)\,dx$ positive or negative?

(b) Is $\int_0^5 f'(x)\,dx$ positive or negative?

(c) Let $g(x) = \int_0^x f(t)\,dt$. Is the graph of g concave up, concave down, or neither near $x = 1$?

(d) Let $h(x) = \int_0^x f'(t)\,dt$. Which is true?

$$h < f \qquad h = f \qquad h > f$$

Problem 7. Fill in the boxes: (Show your work.)

$$\text{if } -2 \leq x \leq 4 \text{ then } \boxed{} \leq 5 - 30x + 51x^2 - 32\,x^3 + 6x^4 \leq \boxed{}.$$

Problem 8. A rectangular box (with a top) is to be constructed from 1200 square units of material. If the base of the box must be a rectangle twice as wide as it is long, how should the box be constructed so as to maximize its volume?

Problem 9. Let $g(x) = \dfrac{x+1}{(x-1)^2}$. Find all inflection points of the graph of g.

Problem 10. The *average value of a function f on the interval* $[a, b]$ is defined to be

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

If a particle is travelling with a velocity of $v(t) = t\sin\left(t^2\right)$ feet per second at time t seconds, what is the particle's average velocity between $t = 0$ and $t = \sqrt{\pi}$ seconds?

EXAM

Final Exam

Math 101

December 15, 2000

- Make sure your solutions are clearly and carefully written.
- Proofread.
- Show your work, but not your scratchwork. Neatness counts.
- You may use a calculator.

Success!