MAT 131 – CALCULUS I – FALL, 2001
FINAL EXAMINATION

December 20, 2001

Name:	**ID Number:**	**Section:**

Problem	1	2	3	4	5	6	7	8	**Total**
Score									

Total possible score: 145 points
Expected Grade Ranges: A 120-145, B 95-119, C 70-94

Problem 1. (10 points) Compute the area that lies under the graph of

$$f(x) = 3 - 2x - x^2$$

and above the x-axis. (**Hint:** First find the points where the graph intersects the x-axis!)

Problem 2. (5 points each) Compute the following definite and indefinite integrals. Check your answers carefully.

(a) $\displaystyle\int_1^3 (x^3 + 4x^2 - 2)dx$

(b) $\displaystyle\int_0^1 \frac{1}{1+t^2}dt$

(c) $\displaystyle\int_0^4 s(\sqrt{s} + 3s)ds$

(d) $\displaystyle\int \sec^2(x)dx$

(e) $\displaystyle\int_{-\pi}^{\pi} \cos\theta d\theta$

(f) $\displaystyle\int \frac{1-2y^2}{1+y^2}dy$

(g) $\displaystyle\int \frac{1}{\sqrt{2s}}ds$

(h) $\displaystyle\int_1^{e^x} \frac{1}{t}dt$

(i) $\displaystyle\int_0^1 e^x dx$

(j) $\displaystyle\int \frac{\sin 2x}{\cos x}dx$

Problem 3. (20 points) Imagine that you are an engineer and being asked to design a cylindrical metal storage bin, open at the top, which contains a given fixed volume V. What is the optimal shape (height and radius) of the bin, so that it takes the least amount of metal to build it?

Problem 4. (10 points) Compute the following limits.

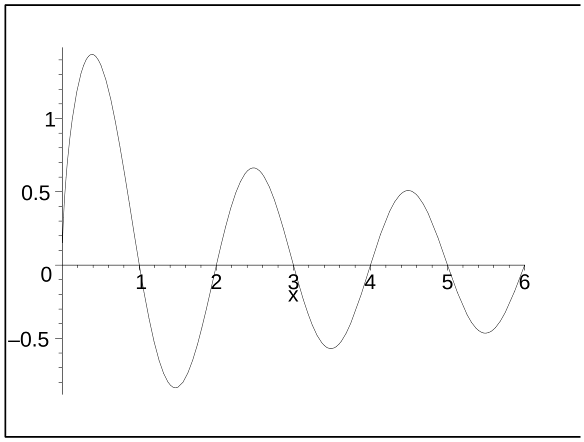
(a) $\lim_{x \to 0} \dfrac{x^2}{1 - e^x}$

(b) $\lim_{\theta \to 0} \dfrac{\sin \theta - \theta}{\theta^2}$

Problem 5. (5 points) Compute the derivative of

$$f(x) = \int_1^x \frac{1 + s^{12} + s^{19} + s^{2001}}{\sqrt{1 + 2s^2 + 4s^4 + 8s^8}} ds.$$

Problem 6. (15 points) Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown.



(a) At what values of x do the local maximum and minimum values occur?

(b) Where does g attain its absolute maximum value?

(c) On what intervals is g concave downward?

Problem 7. (15 points) Imagine that a cook is making a very large round pancake. As the batter pours onto the pan, the area of the pancake is increasing at a constant rate of 3 square inches per second.

(a) What is the area of the pancake t seconds after the cook begins pouring the batter?

(b) Give a formula for the rate of increase of the radius of the pancake t seconds after the cook begins pouring the batter.

(c) When the pancake has an area of 12 square inches, how fast is the radius increasing?

Problem 8. (20 points) Let

$$f(x) = \frac{\sin x + x}{e^x}.$$

(a) Find the equation of the line tangent to the graph of $f(x)$ at $x = 0$.

(b) Find the differential df.

(c) Use one of these to approximate $f(.03)$.