

**MAT131 Calculus I Fall, 2000**  
**Final Exam, Dec. 20, 2000, 2:00-4:30 P.M.**

Name: \_\_\_\_\_ ID: \_\_\_\_\_  
 Section: \_\_\_\_\_ Section Teacher: \_\_\_\_\_

If you are not sure of your section, consult the table below.

Section	Instructor	Time
1	Mieczkowski	Tu Th 2:20
2	Mieczkowski	Tu Th 3:50
3	Kim	M W 5:30
4	Krol	M W 2:15
5	Lundberg	M W 8:20
6	Coffee	M W 10:30
7	Kim	M W 8:30
9	Buse	Tu Th 8:20
10	Yau	M W 3:20
11	Friedman	Tu Th 9:50
12	Friedman	Tu Th 12:50
13	Sporn	Tu Th 8:20
14	Yau	M W 2:15
15	Behrstock	Tu Th 11:20
16	Behrstock	Tu Th 3:50
17	Rasdeaconu	Tu Th 7:00
18	Buse	Tu Th 8:20

Problem	Points	Grade
1	8	
2	9	
3	10	
4	9	
5	10	
6	13	
7	9	
8	9	
9	10	
Total	87	

*WORK all problems on these pages.  
 SHOW all work you want graded.  
 WRITE CAREFULLY: points may  
 be taken off for meaningless state-  
 ments.*

1. a) Evaluate:

$$\lim_{h \rightarrow 0} \frac{(2+h)^2 - 4}{h}$$

- b) Find a function  $f(x)$  and a number  $x_0$  such that the expression in part a) is the derivative of  $f(x)$  at  $x_0$ .

2. Suppose a ball is dropped from the top of a building whose height is 80 feet. Assuming that its distance from the top is  $s(t) = 16t^2$  after  $t$  seconds, how fast is it descending when  $t = 3$  seconds? How fast is it descending when it hits the ground? How long does it take the ball to hit the ground?

3. Let  $\lfloor x \rfloor$  denote the largest integer that is less than or equal to  $x$ .

a) Find

$$\lim_{x \rightarrow 2^+} \lfloor x \rfloor.$$

b) Find

$$\lim_{x \rightarrow 2^-} \lfloor x \rfloor.$$

c) For what values of  $x$  is  $\lfloor x \rfloor$  continuous?

4. Find the number at which the function  $f(x) = x^2 + 3x$  has a minimum and find the value of  $f(x)$  at that number.

Draw the graph of  $f(x)$  for  $-2 \leq x \leq 2$

5. a) Draw the graph of  $g(x) = x^3 - x$  for  $-2 \leq x \leq 2$ .

b) Find all local minima, local maxima and inflexion points of  $g(x)$  and mark them on the graph.

6. Let the function  $f(x)$  be given by

$$f(x) = \begin{cases} 0, & x \leq 0 \\ x, & 0 \leq x \leq 2, \\ 2, & x \geq 2. \end{cases}$$

a) Find the value of the function

$$\Phi(x) = \int_0^x f(t) dt$$

for each  $x \leq 0$ , for each  $x$  between 0 and 2, and then for each  $x \geq 2$ .

b) Draw the graph of  $\Phi(x)$  for  $-1 \leq x \leq 3$

c) Compute the derivative of  $\Phi(x)$ .

Find the difference between  $f(x)$  and  $\Phi(x)$

7. Let  $f(x) = x^{1/4}$ .

a) Find  $f'(x)$ .

b) Find the equation of the tangent line to the graph of  $f$  at  $x = 16$ .

c) Use a linear approximation to  $f$  to find an approximate value for  $17^{1/4}$ .

8) Compute the following anti-derivatives (or indefinite integrals):

a)  $\int(3x^3 - 3x + 3)dx$

b)  $\int(1/x - 1/x^2)dx$

c)  $\int(e^x - \sin x)dx$

9) Assume that a 10-foot ladder is leaning against a vertical wall with its base 2 feet from the wall. The base starts to slip and when it is 6 feet from the wall, it is moving away from the wall at 8 feet per second. How fast is the top of the ladder descending at this time? Show clearly what you did to get your answer.