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**EXAM**

Test 2

Math 118

November 13, 2001

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**ANSWERS**

**Problem 1.** A woman has \$75 deducted from her paycheck at the end of each month and put into a savings account earning 9% interest compounded monthly. She continues these deposits for 10 years.

(a) How much will the account be worth after 10 years?

*Answer:*

We use

$$F = D \left( \frac{(1 + \frac{r}{n})^{tn} - 1}{\frac{r}{n}} \right),$$

where  $D = 75$ ,  $n = 12$ ,  $t = 10$ , and  $r = .09$ . This gives

$$\begin{aligned} F &= \$75 \left( \frac{(1 + \frac{.09}{12})^{120} - 1}{\frac{.09}{12}} \right) \\ &= \$75 \left( \frac{(1.075)^{120} - 1}{.0075} \right) \\ &= \$14513.57 \end{aligned}$$

**Problem 1.** *Continued.*

(b) How much of the future value will be from deposits?

*Answer:*

There will be 120 deposits of \$75 each, for a total of \$9000.

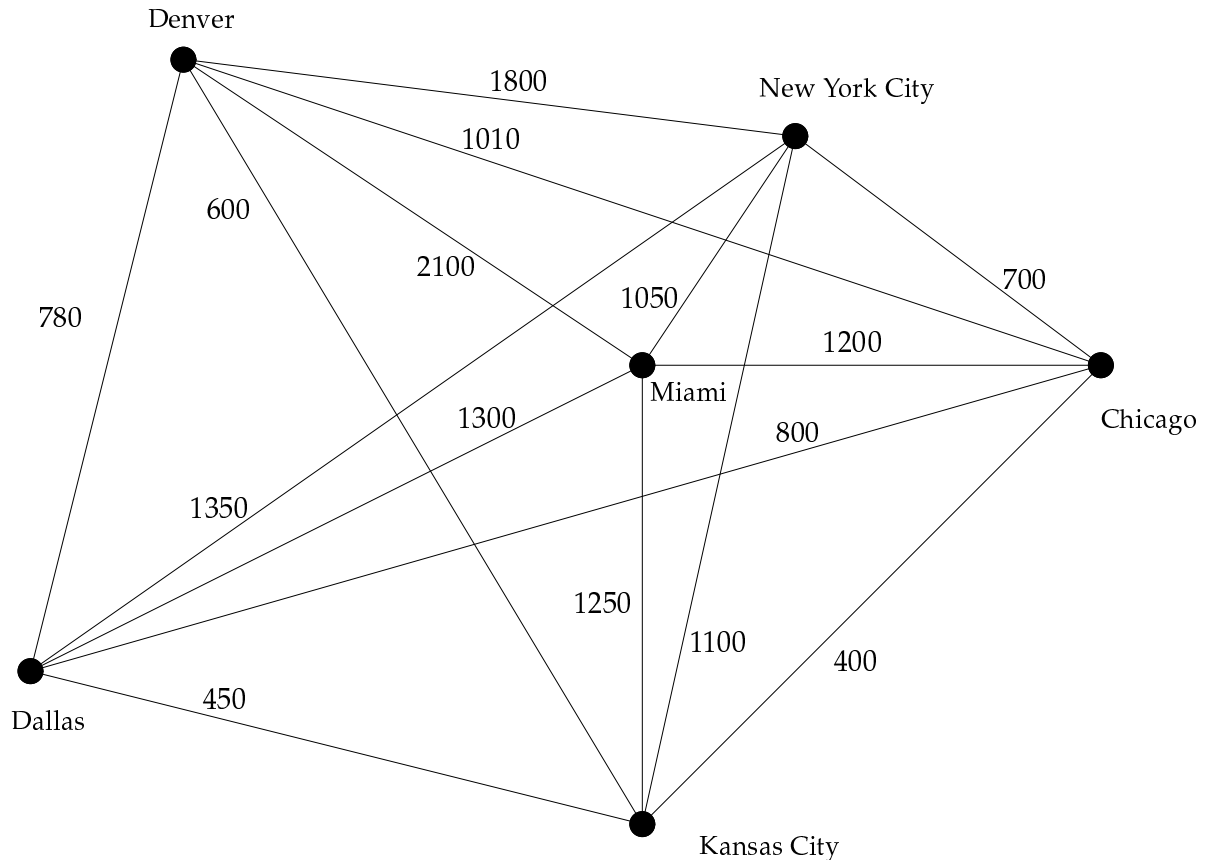
(c) How much of the future value will be from interest?

*Answer:*

The remaining  $\$14513.57 - \$9000 = \$5513.57$  is from interest.

**Problem 2.** Suppose that you need to plan a trip that includes stops in New York, Denver, Chicago, Kansas City, Dallas, and Miami. Assume that you live in one of these cities, so that your planned trip must begin and end in the same city. You would like your trip to be as short as possible.

I checked the distances between each of these cities and came up with the following weighted graph:



- (a) Use the greedy algorithm to find a Hamiltonian circuit. What is the total distance of this circuit?

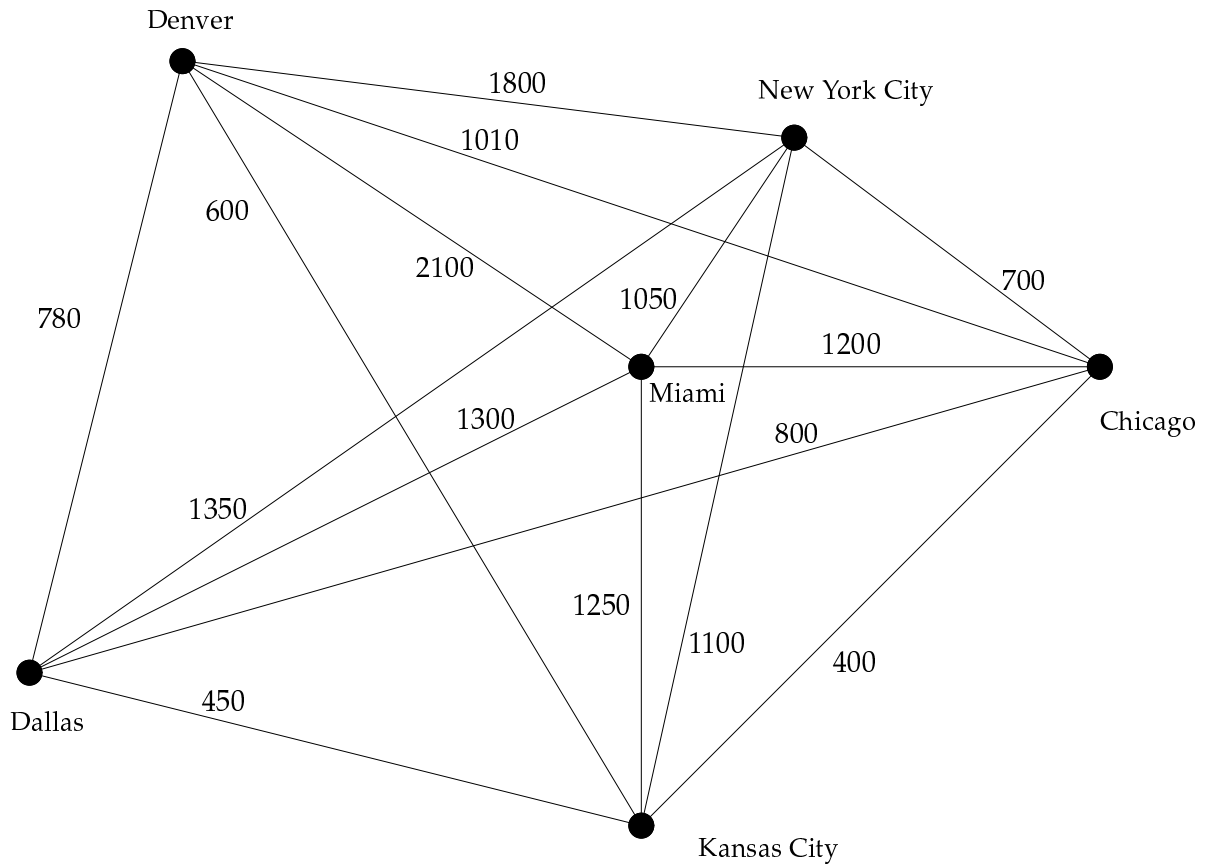
*Answer:*

We pick the edges:

- Chicago – Kansas City (400)
- Dallas – Kansas City (450)
- New York – Chicago (700)
- Denver – Dallas (780)
- Denver – Miami (2100)
- Miami – New York (1050)

The total length is 5480 miles.

**Problem 2.** *Continued.* Here is another picture of the same graph.



- (b) Starting with the Denver vertex, use the nearest neighbor algorithm to find a Hamiltonian circuit. What is the total distance of this circuit?

*Answer:*

We begin at Denver and produce this circuit:

Denver  $\rightarrow$  Kansas City  $\rightarrow$  Chicago  $\rightarrow$  New York  $\rightarrow$  Miami  $\rightarrow$  Dallas  $\rightarrow$  Denver

The distance of this route is  $600 + 400 + 700 + 1050 + 1300 + 780 = 4830$ .

**Problem 3.**

(a) What is the diameter of  $K_{3,3}$ ?

*Answer:*

2. If two vertices are not connected by an edge, then you can connect them by a path of length two by passing through one of the other vertices.

(b) Does  $K_{3,3}$  have any Euler paths?

*Answer:*

No. All six vertices have degree three, which is odd.

(c) Does  $K_{5,2}$  have any Euler paths?

*Answer:*

Yes.  $K_{5,2}$  has five vertices of degree 2 and two vertices of degree 5. There will exist an Euler path starting at one of the odd vertices and ending at the other odd vertex.

**Problem 3.**

(c) Who solved the Bridges of Königsburg problem? Multiple choice:

- Leonard Euler
- Kenneth Appel and Wolfgang Haken
- William Hamilton
- Bernhard Riemann
- Paul Seymour and Neil Robertson

(d) If I embed  $K_6$  on a surface in such a way that it has seven faces, then which surface must it be? Circle one:

*Answer:*

$K_6$  has 6 vertices and 15 edges. If it is embedded on a surface so that the graph has seven faces, then the Euler number of the surface is  $6 - 15 + 7 = -2$ . Therefore the surface has genus two.