
EXAM

Test 1

Math 118

October 2, 2001

ANSWERS

Problem 1. The top three candidates for New York city mayor are democrats Mark Green and Fernando Ferrer, and republican Michael Bloomberg. Suppose that 21 voters were asked about their preferences between these three candidates and current mayor Rudolph Giuliani. Use the following data to complete this problem.

| | | | | | | | | | | | |
|-----------|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 2 | 1 | 2 | 2 | 3 | 3 | 1 | 2 | 2 |
| Bloomberg | 1 | 1 | 2 | 4 | 2 | 4 | 4 | 3 | 4 | 3 | 2 |
| Ferrer | 4 | 3 | 3 | 1 | 1 | 3 | 2 | 1 | 3 | 2 | 4 |
| Giuliani | 2 | 2 | 1 | 3 | 4 | 2 | 3 | 4 | 1 | 1 | 3 |
| Green | 3 | 4 | 4 | 2 | 3 | 1 | 1 | 2 | 2 | 4 | 1 |

- (a) Suppose that simple plurality will be used to determine a winner from these four candidates. Which candidate will win?

Answer:

Bloomberg gets 3 votes, Ferrer gets 6, Giuliani gets 5, and Green gets 7. Green wins the plurality election.

- (b) Suppose that Borda's method will be used to determine a winner from these four candidates. Which candidate will win?

Answer:

We compute the Borda counts:

$$\text{Bloomberg: } 3 \times 4 + 6 \times 3 + 5 \times 2 + 7 \times 1 = 47$$

$$\text{Ferrer: } 6 \times 4 + 5 \times 3 + 7 \times 2 + 3 \times 1 = 56$$

$$\text{Giuliani: } 5 \times 4 + 5 \times 3 + 6 \times 2 + 5 \times 1 = 52$$

$$\text{Green: } 7 \times 4 + 5 \times 3 + 3 \times 2 + 6 \times 1 = 55$$

Ferrer wins by Borda's method.

Problem 1. Continued.

- (c) Actually, there will be a runoff election to decide which candidate will run on the democratic ticket. Decide whether Green or Ferrer will win a runoff election.

Answer:

Using the preference data, we see that in a runoff election between Ferrer and Green, Ferrer will defeat Green 12 votes to 9.

| | | | | | | | | | | | |
|--------|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 2 | 1 | 2 | 2 | 3 | 3 | 1 | 2 | 2 |
| Ferrer | 4 | 3 | 3 | 1 | 1 | 3 | 2 | 1 | 3 | 2 | 4 |
| Green | 3 | 4 | 4 | 2 | 3 | 1 | 1 | 2 | 2 | 4 | 1 |

- (d) Use plurality to decide who will win the three candidate election between the winner of the democratic runoff, Bloomberg, and Giuliani?

Answer:

After removing Green, Bloomberg gets 5 votes, Ferrer gets 9, Giuliani gets 7. So, Ferrer wins.

| | | | | | | | | | | | |
|-----------|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 2 | 1 | 2 | 2 | 3 | 3 | 1 | 2 | 2 |
| Bloomberg | 1 | 1 | 2 | 4 | 2 | 4 | 4 | 3 | 4 | 3 | 2 |
| Ferrer | 4 | 3 | 3 | 1 | 1 | 3 | 2 | 1 | 3 | 2 | 4 |
| Giuliani | 2 | 2 | 1 | 3 | 4 | 2 | 3 | 4 | 1 | 1 | 3 |

Problem 1. Continued.

Even though many voters would like to see Giuliani elected again, term limitation legislation enacted in 1993 prohibits Rudy Giuliani from serving an additional term. So, now assume that Giuliani is removed from the race.

- (c) Who will win the election between the winner of the democratic runoff and Michael Bloomberg?

Answer:

After Giuliani is removed and Ferrer wins the democratic ticket, we have an election between Ferrer and Bloomberg:

| | | | | | | | | | | | |
|-----------|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 2 | 1 | 2 | 2 | 3 | 3 | 1 | 2 | 2 |
| Bloomberg | 1 | 1 | 2 | 4 | 2 | 4 | 4 | 3 | 4 | 3 | 2 |
| Ferrer | 4 | 3 | 3 | 1 | 1 | 3 | 2 | 1 | 3 | 2 | 4 |

Ferrer defeats Bloomberg 14 to 7.

- (d) Can any of the voters who ranked Giuliani first vote strategically in order to obtain a preferential outcome?

Answer:

No. It is not possible for the Giuliani supporters to change their votes in such a way that Bloomberg wins. This group of voters *can* affect the outcome of the election by voting for Green over Ferrer. This would result in Green defeating Ferrer in the democratic race and Green defeating Bloomberg in the final election. However, (all but one of) Giuliani's supporters would find this outcome less preferable than Ferrer's victory.

Problem 2. A woman deposits \$600 into a savings account earning 5% interest compounded monthly.

(a) What will be the total amount in the account in 5 years?

Answer:

We use $F = P \left(1 + \frac{r}{n}\right)^{nt}$:

$$F = \$600 \left(1 + \frac{.05}{12}\right)^{(5)(12)} = \$770.02.$$

(b) How long will it take for the account to reach \$2000?

Answer:

We use $F = P \left(1 + \frac{r}{n}\right)^{nt}$:

$$\begin{aligned} \$2000 &= \$600 \left(1 + \frac{.05}{12}\right)^{12t} \\ \frac{2000}{600} &= \left(1 + \frac{.05}{12}\right)^{12t} \\ \log \frac{2000}{600} &= \log \left(\left(1 + \frac{.05}{12}\right)^{12t} \right) \\ \log \frac{2000}{600} &= 12t \log \left(\left(1 + \frac{.05}{12}\right) \right) \\ 0.52287875 &= (12)(.0018058009)t \\ t &= 24.129599 \end{aligned}$$

Problem 3. Construct preference ballots for an election with ten voters and three candidates, call them A , B , and C , satisfying the following properties:

- A wins a simple plurality election.
- B is the Condorcet winner.

Here is a table to help organize your answer: write numbers of voters above each possible preference ranking so that the sum is ten (you may want to use zeroes).

Answer:

| | | | | | | |
|-----|---|---|---|---|---|---|
| | 4 | 0 | 0 | 3 | 0 | 3 |
| A | 1 | 1 | 2 | 3 | 2 | 3 |
| B | 2 | 3 | 1 | 1 | 3 | 2 |
| C | 3 | 2 | 3 | 2 | 1 | 1 |

A wins the election 4 to 3 to 3. But, in head to head comparisons, B beats A 6 to 4 and B beats C 7 to 3. So, B is the Condorcet winner.

Problem 4. Multiple choice: Suppose I deposit \$100 every month into an account bearing 11% interest compounded daily. After 2 years, the amount in the account will be

- (a) Less than \$111.12
- (b) Between \$111.12 and \$2400.00
- (c) Between \$2400 and \$2990.49
- (d) Between \$2987.59 and \$5980.97
- (e) Greater than \$5975.18.

Answer:

24 deposits of \$100 each will result in \$2400, so the amount in the account will be at least \$2400. If we deposited all \$2400 in the account at the beginning, we would earn interest on the whole amount, which by the compound interest formula is \$2990.49. This amount certainly exceeds the amount in the account after two years of monthly deposits.

So, the actual amount earned will be between \$2400 and \$2990.49.