

Problem 1. Here is a table of four cities A, B, C, D and the mileage between them.

	A	B	C	D
A		5	9	6
B	5		3	2
C	9	3		5
D	6	2	5	

Sketch a weighted graph representing these four cities and the distance between them.

- Use the nearest neighbor algorithm (starting with vertex A) to find a Hamiltonian circuit. What is its length?
- Use the greedy algorithm (starting with vertex A) to find a Hamiltonian circuit. What is its length?
- Find the actual solution to the travelling salesman problem for this graph.

Problem 2. If I draw a diameter three graph of size twenty without any crossings, how many faces will it have?

Problem 3. Circle the one that yields the larger account:

- Making monthly deposits of \$100 at 20% interest for 10 years.
- Making monthly deposits of \$200 at 10% interest for 10 years.
- Making monthly deposits of \$100 at 10% interest for 20 years.

Problem 4. Do exercises 11 and 12 in section 1.2.

Problem 5. Do exercise 18 in section 6.1.

Problem 6. Use the Euclidean algorithm to find the greatest common divisor of 1001 and 245. Then, find integers x and y so that

$$\gcd(1001, 245) = x \cdot 1001 + y \cdot 245.$$

Problem 7. Do exercises 37 in section 3.3 and 19 in section 3.4.

Problem 8. What is the sum?

$$1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots + \left(\frac{1}{2}\right)^{86} + \left(\frac{1}{2}\right)^{87}$$

Problem 9. For an upcoming election involving candidates Alice, Bob, and Chris, a preliminary poll is taken and the following voter preference rankings are collected. The numbers at the top indicate the percentage of people polled.

	9	28	11	23	23	6
Alice	1	1	2	3	2	3
Bob	2	3	1	1	3	2
Chris	3	2	3	2	1	1

- (a) Is there a Condorcet winner?
- (b) If we assume that voters approve of the candidate ranked first and second, which candidate would win if the winner was decided by an approval vote?

Problem 10. True or false:

- (a) Every election has a Condorcet winner.
- (b) 323 is prime.
- (c) The greedy algorithm always produces a shorter Hamiltonian circuit than the nearest neighbor algorithm.
- (d) $-136 \bmod 7 = 4$.
- (e) The equation $x^2 + y^2 = z^2$ has many integer solutions, but the equation $a^3 + b^3 = c^3$ has no nonzero integer solutions.
- (f) The plurality method of deciding an election, with a runoff between the top two candidates, can violate the independence from irrelevant alternatives property.
- (g) The Euler characteristic of a torus equals one.
- (h) All planar 3-regular graphs of size 12 have diameter greater than 3.
- (i) Borda's method of deciding an election may violate the Pareto optimality condition.
- (j) There exist planar maps that cannot be colored with four colors, but they have hundreds of territories.

EXAM

Practice Exam

Math 118

December 8, 2001
