MAT118 Spring 2012 Midterm 1

Name: _____ SB ID number: _____

Please circle the number of your recitation.

1. W 5:20 – SB Union Atyam	**2.** M 12:50 – EarthSpaceSci Atyam	**3.** Th 12:50 – Physics Gallardo

********** **DO NOT WRITE BELOW THIS LINE.** **********

Problem 1: _____ /25	**Problem 2**: _____ /25	**Problem 3**: _____ /25

Problem 4: _____ /25

TOTAL: _____ /100

Instructions: The exam is closed book, closed notes, calculators are not allowed, and all cell phones and other electronic devices must be turned off for the duration of the exam. You will have approximately 50 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please **show all work**, unless instructed otherwise. Partial credit will be given only for work shown. You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., then **please raise your hand**.

There is a **SCRATCH PAGE** at the end of the packet which you may use.

Name: _____ **Problem 1**: _____ /25

Problem 1(25 points) The following table gives the **preference schedule** for an election with four candidates.

Number of voters	6	5	4
1^{st} place	A	C	D
2^{nd} place	B	B	B
3^{rd} place	C	A	A
4^{th} place	D	D	C

(a)(5 points) Which candidate wins under the **plurality-with-elimination** method (sometimes also called "instant runoff")? **Show all your work.**

(b)(10 points) This election has a **Condorcet candidate**. Find the Condorcet candidate, and state whether or not the Condorcet criterion is satisfied. **Show all your work.**

(c)(10 points) Candidate C drops out of the race, but otherwise all relative rankings remain the same. Determine the new winner under plurality-with-elimination, and state which fairness criterion is violated by this outcome. **Show all your work.**

Name: _____ **Problem 2**: _____ /25

Problem 2(25 points) In a law firm with one founder, F, two junior partners, P and Q, and one associate, A, a winning coalition is made up of either the founder and at least one junior partner, or both junior partners and the associate. Here is the complete list of all **winning coalitions**.

$$\{F, P, Q, A\},\ \{F, P, Q\},\ \{F, P, A\},\ \{F, Q, A\},\ \{P, Q, A\},\ \{F, P\},\ \{F, Q\}.$$

(a)(5 points) In each winning coalition listed above, determine every **critical player**. Indicate your answer clearly by underlining in the above list or by listing winning coalitions and critical players below. You need not show work for this part.

(b)(10 points) Compute the **Banzhaf number** of each player, compute the total, and then compute the **Banzhaf index** of each player. Leave the Banzhaf index in the form of a fraction. **Show all your work.**

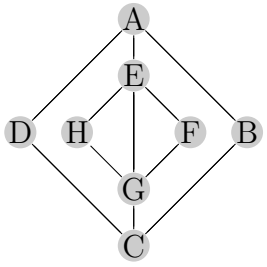
B_F:___, B_P:___, B_Q:___, B_A:___, Total:___, β_F:___, β_P:___, β_Q:___, β_A:___.

(c)(10 points) Compute the **Shapley-Shubik number** of each player, compute the total, and then compute the **Shapley-Shubik index** of each player (left as a fraction). You may compute this either by listing all **sequential coalitions** together with **pivotal players** or by counting the number of sequential coalitions associated to every winning coalition with specified critical player. Either way, **show all your work.**

SS_F:___, SS_P:___, SS_Q:___, SS_A:___, Total:___, σ_F:___, σ_P:___, σ_Q:___, σ_A:___.

Name: _____ **Problem 3**: _____ /25

Problem 3(25 points)



(a)(5 points) For the graph above, list the **degrees** of all eight vertices.

A:____, B:____, C:____, D:____, E:____, F:____, G:____, H:____.

(b)(5 points) List all **odd vertices**, and also state the total number of odd vertices.

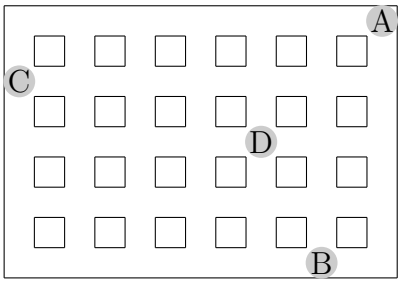
(c)(5 points) State whether not this graph has an **Euler cycle**, including a justification (if you use a result from the book, that is adequate justification, but you should give the correct statement of the result).

(d)(5 points) State whether not this graph has an **Euler path**, including a justification (if you use a result from the book, that is adequate justification, but you should give the correct statement of the result). Recall that in our definition, the start vertex of the Euler path is always different from the stop vertex. In case there is an Euler path, also list the vertices which will be the start and stop.

(e)(5 points) Find an **optimal Eulerization** of this graph. List the existing edge or existing edges which should be doubled in your optimal Eulerization.

Name: _____ **Problem 4**: _____ /25

Problem 4(25 points)



The grid above shows the streets in a taxi driver's zone. The driver must find a **Hamiltonian cycle** beginning and ending at the intersection A, which crosses the intersections B, C and D, and which traverses the shortest distance both horizontally plus vertically (each small square is a city block).

(a)(5 points) Fill in the following distance chart, where units are city blocks travelled (horizontally plus vertically).

	A	B	C	D
A	*			
B		*		
C			*	
D				*

(b)(10 points) Find one Hamiltonian cycle using the **nearest neighbor** method beginning at A, and compute the total length of this cycle. **Show all your work.**

(c)(10 points) Find one Hamiltonian cycle using the **nearest neighbor** method beginning at C, and compute the total length of this cycle. **Show all your work.**

SCRATCH PAGE