MAT118 Spring 2012 Final Exam

Name: ______________________________ SB ID number: ______________________

Please circle the number of your recitation.

1. W 5:20 – SB Union Atyam	**2.** M 12:50 – EarthSpaceSci Atyam	**3.** Th 12:50 – Physics Gallardo

*************************** **DO NOT WRITE BELOW THIS LINE.** *************

Problem 1: ______ /30 **Problem 2**: ______ /30 **Problem 3**: ______ /30

Problem 4: ______ /30 **Problem 5**: ______ /30 **Problem 6**: ______ /25

Problem 7: ______ /25

TOTAL: _________ /200

Instructions: The exam is closed book, closed notes, calculators are not allowed, and all cell phones and other electronic devices must be turned off for the duration of the exam. You will have approximately 150 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please **show all work**, unless instructed otherwise. Partial credit will be given only for work shown. You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., then **please raise your hand**.

There is a **SCRATCH PAGE** at the end of the packet which you may use.

Common Formulas

Future value under simple interest after T years at APR r.

$$F = P(1 + rT).$$

Future value under compound interest compounded m times annually after T years at APR r.

$$F = P(1 + (r/m))^{mT}.$$

Annual percentage yield for compound interest of APR r compounded m times annually.

$$\text{APY} = (1 + (r/m))^m - 1.$$

The sum of T terms in a geometric sequence with growth factor $C > 1$ whose initial term is P.

$$S = P(C^n - 1)/(C - 1).$$

Logistic growth model with carrying capacity C and growth parameter r.

$$P_{N+1}/C = r(1 - (P_N/C))(P_N/C),\ p_{n+1} = r(1 - p_n)p_n.$$

Number ${}_r\,P_n$ of ordered r-tuples of distinct objects from among n total objects.

$${}_r\,P_n = n!/(n-r)!$$

Number ${}_r\,C_n$ of unordered r-tuples of distinct objects from among n total objects.

$${}_r\,C_n = n!/[r!(n-r)!]$$

The first few terms of Pascal's triangle.

$$
\begin{array}{ccccccccccccc}
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 & & 1 & & 4 & & 6 & & 4 & & 1 & & \\
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1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1
\end{array}
$$

Name: ______________________________ **Problem 1**: ______________ /30

Problem 1(30 points) The following table gives the **preference schedule** for an election with four candidates and twenty-seven voters.

Number of voters	10	6	5	4	2
1st place	A	B	C	D	D
2nd place	C	D	B	C	A
3rd place	D	C	D	B	B
4th place	B	A	A	A	C

(a)(10 points) Which candidate wins under the **plurality-with-elimination** method (sometimes also called "instant runoff")? **Show all your work.**

(b)(10 points) This election has a **Condorcet candidate**. Find the Condorcet candidate, and state whether or not the Condorcet criterion is satisfied. **Show all your work.**

(c)(10 points) Candidate D drops out of the race, but otherwise all relative rankings remain the same. Determine the new winner under plurality-with-elimination, and state which fairness criterion is violated by this outcome. **Show all your work.**

Name: ____ **Problem 2**: _________________ /30

Problem 2(30 points) In a law firm the founder, F, has 3 votes, the senior partner, P, has 2 votes, and the two junior partners, J and K, each have 1 vote, for a total of $3+2+1+1=7$ votes. The quota for approving a decision is 5 votes.

(a)(10 points) Fill in the following table of all **winning coalitions**. In each winning coalition, underline any and all **critical players**. In each row, tally the number of times each player is a critical player in a winning coalition in that row.

r	Winning coalitions with r players	F	P	J	K
1	-	0	0	0	0
2					
3					
4					

(b)(5 points) Is any player a **dummy**? If yes, list any and all players which are dummies. Does any player have **veto power**? If yes, list any and all players who have veto power?

Any dummy? __ Yes, __ No. If yes, list any and all dummies: ______.

Any veto power? __ Yes, __ No. If yes, list any and all veto players: ______.

(c)(5 points) Compute the **Banzhaf number** of each player, compute the total, and then compute the **Banzhaf index** of each player. Leave the Banzhaf index in the form of a fraction. **Show all your work.**

B_F:___, B_P:___, B_J:___, B_K:___, Total:____, β_F:____, β_P:____, β_J:____, β_K:____.

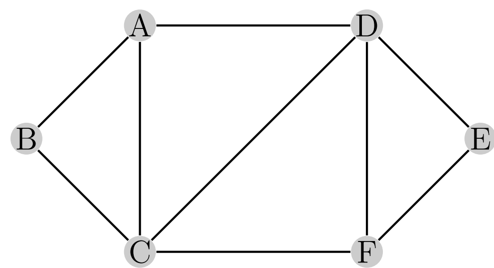
Name: ___________________________________ **Problem 2, continued**

(d)(10 points) Compute the **Shapley-Shubik number** of each player, compute the total, and then compute the **Shapley-Shubik index** of each player (left as a fraction). You may compute this either by listing all **sequential coalitions** together with **pivotal players** or by counting the number of sequential coalitions associated to every winning coalition with specified critical player. Either way, **show all your work.**

SS_F:___, SS_P:___, SS_J:___, SS_K:___, Total:____, σ_F:____, σ_P:____, σ_J:____, σ_K:____.

Name: ______________________________ **Problem 3**: ______________ /30

Problem 3(30 points)



(a)(5 points) For the graph above, list the **degrees** of all six vertices.

A:____, B:____, C:____, D:____, E:____, F:____.

(b)(5 points) List all **odd vertices**, and also state the total number of odd vertices.

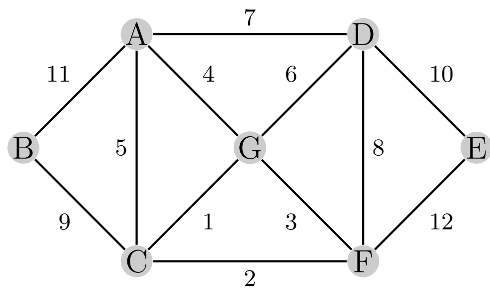
(c)(5 points) State whether not this graph has an **Euler cycle**, including a justification (if you use a result from the book, that is adequate justification, but you should give the correct statement of the result).

(d)(10 points) State whether not this graph has an **Euler path**, including a justification (if you use a result from the book, that is adequate justification, but you should give the correct statement of the result). Recall that in our definition, the start vertex of the Euler path is always different from the stop vertex. In case there is an Euler path, also list the vertices which will be the start and stop.

(e)(5 points) Find an **optimal Eulerization** of this graph. List the existing edge or existing edges which should be doubled in your optimal Eulerization.

Name: ______________________________________ **Problem 4**: ________________ /30

Problem 4(30 points)



The weights in the above graph are the costs of edges. Apply **Kruskal's algorithm** to find a **minimal spanning tree**.

(a)(25 points) In the box below, list the edges of the minimal spanning tree in the order they are produced by Kruskal's algorithm (for each edge, it is immaterial which of the two vertices you list first).

(b)(5 points) Finally, sum up the total cost for your minimal spanning tree.

Edge	1^{st} Vertex	2^{nd} Vertex	Cost
1^{st}			
2^{nd}			
3^{rd}			
4^{th}			
5^{th}			
6^{th}			

Total cost______

Name: ______________________________ **Problem 5**: ____________ /30

Problem 5(30 points) The stock of a pod-racing company on Tatooine experiences annual growth, from the beginning of each year to that end of that same year, of APR $33.333\ldots\%$, i.e., the equivalent percentage of the fraction $1/3$.

(a)(10 points) Jabba owns $P = \$270,000$ stock at the beginning of the first year (in "wuipiupi", the local currency). Find the future value F_1 of Jabba's stock after 1 year as a whole number of dollars. Also find the **growth factor** $C = F_1/P$. Write C as a fraction in lowest terms. **Show all your work.**

Future value F_1 ________ **Growth factor** C ________

(b)(15 points) In the table below, compute the future value of Jabba's stock after T years for each value of T. You may leave your answer as a fraction (proper or improper) or express your answer in terms of whole dollars. **Do not express your answer in terms of an unevaluated power of a decimal. Show all work.**

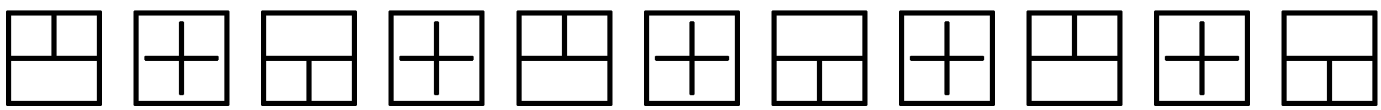
Years T	0	1	2	3
Future Value F_T				

(c)(5 points) Give the formula for the value of Jabba's stock after T years (where T is an unspecified positive whole number). Please express your answer in terms of fractions. The only variable or unevaluated constant in your formula should be T. **Show all your work.**

Future Value F_T __________

Name: ________________________________ **Problem 6**: ______________ /25

Problem 6(25 points) The following border pattern extends infinitely both left and right. For the following questions, draw **and label** the specified points and lines directly on the border pattern.



(a)(5 points) Is there a non-identity translation symmetry? If yes, then label a vector $\vec{w}$ next to the pattern indicating the smallest nonzero translation vector which is a symmetry.

___ Yes, ___ No.

(b)(5 points) Is there a horizontal reflection symmetry? If yes, then label in the pattern a horizontal line H which is an axis for the reflection symmetry.

___ Yes, ___ No.

(c)(5 points) Is there a vertical reflection symmetry? If yes, then label in the pattern a vertical line V which is an axis for the reflection symmetry.

___ Yes, ___ No.

(d)(5 points) Is there a non-identity rotational symmetry? If yes, then label in the pattern a rotocenter C for a rotation symmetry, and list the nonzero angle of the symmetry below.

___ Yes, ___ No, If yes, angle of symmetry: ______.

(e)(5 points) Is there a glide reflection symmetry? If yes, then label in the pattern a horizontal line L which is an axis for the glide reflection symmetry, and also specify below the smallest nonzero fraction of $\vec{w}$ which is the translation vector for a glide symmetry.

___ Yes, ___ No, If yes, fraction of $\vec{w}$: ______.

Name: ______________________________ **Problem 7**: ____________ /25

Problem 7(25 points) A certain probabilistic process consists of flipping a fair coin six times in a row.

(a)(5 points) Compute the total number N of possible outcomes of this probabilistic process, i.e., the total size of the sample space. **Show all your work.**

$$N = \underline{\qquad}.$$

(b)(5 points) In this equiprobable probability space, compute the probability p of each outcome. Leave your answer in the form of a fraction. **Show all your work.**

$$N = \underline{\qquad}.$$

(c)(10 points) Consider the event E that there are precisely 4 heads and 2 tails among the 6 coin flips. Compute the total number m of outcomes for which this event is true. There is more than one way to compute this, and you may use any correct method. However you mush **show all your work.**

$$m = \underline{\qquad}.$$

(d)(5 points) Compute the total probability P of the event E. Leave your answer in the form of a fraction in least terms.

$$P = \underline{\qquad}.$$

SCRATCH PAGE