

Problem Set 12

Disclaimer For open-ended problems, part of the problem is to give a precise formulation. Especially for the problems in Part II, you should do as much of the problems as is useful to you. For each problem, it is important you understand how to verify all details. However, if you are pressed for time, you may write-up only the most important steps, instead of every detail. As an adult, you are expected to use your judgment in interpreting what this precisely means.

Late homework policy. Late work will be accepted only with a medical note or for another Institute-approved reason.

Cooperation policy. You are strongly encouraged to work with others, but the final write-up must be entirely your own and based on your own understanding.

Part I. These problems are from the textbook. You are expected to read *all* the problems from the sections of the textbook covered that week. You are asked to write-up and turn-in only the problems assigned below.

Part II. These problems are not necessarily from the textbook. Often they will be exercises in commutative algebra, category theory, homological algebra or sheaf theory.

Part I(25 points)

- (a) (25 points) p. 232, Section III.5, Problem 5.7

Part II(25 points)

Problem 1(25 points) Do Exercise 5.8 of §III.5 on p. 232. The argument could be somewhat simpler; you may use any correct argument you like. The hypothesis that the field is algebraically closed is unnecessary.

Extra credit(10 points) Let A be Noetherian ring, and let $f : X \rightarrow \text{Spec } A$ be a proper morphism such that every irreducible component of every fiber has dimension 0 or 1, i.e., the relative version of Exercise 5.8. Is the morphism f projective? What if we assume $A = \mathbb{C}[[x]]$?