Practice Midterm 1

MAT 118

Feb 26, 2005

Name: (please print)	**ID #:**

Directions: There are 4 problems on 4 pages in this exam. Do all of your work in this exam booklet, and cross out any work that should be ignored. **Show your reasoning and computations — not just the answer.** You may not use any books, notes, calculators, or discussions with friends during this exam.

1. The eight members of a board of directors of an art museum must select a new museum head. There are three candiates: Zukoff (Z), Perella (P) and Mintz (M). The preference rankings of the board members are included in the table.

Number of voters	2	2	3	4
1^{st} place	Z	P	P	M
2^{nd} place	M	Z	M	P
3^{rd} place	P	M	Z	Z

(a) Which candidate would win a plurality election?

(b) Which candidate would win a plurality election with a runoff between the top two finishers?

(c) Which candidate would win the Borda count?

(d) Which candidate, if any, is the Condorcet winner?

(e) Can the voters of the last group (those who ranked Mintz first) get a better result in Borda count method by voting strategically?

2. Consider the weighted voting system $[8 : 6, 4, 2, 1]$.

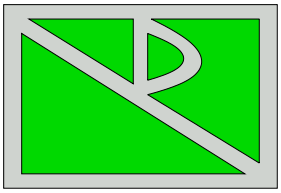
(a) Find the Banzhaf power index of each player.

(b) Are there any players who are dictators? have veto power? dummies? Justify your answer.

(c) Assume that player P_4 buys one vote from player P_1, so the new weights are $[8 : 5, 4, 2, 2]$. Find the new Banzhaf power index.

(d) Which would give player P_4 more power: buying one vote from P_1 or buying one vote from player P_2?

3. The sidewalks of a park are shown in the figure on the right (they go around the outside, across the middle, and make the D-shape). A city worker needs to sweep the walks, and wants to cover each one exactly once, without retracing his steps or walking on the grass.



(a) Draw the graph corresponding to this situation.

(b) Is it possible for the worker cover each part of the walk exactly once, ending up where he started? (That is, does the graph have an Eulerian circuit?) Justify your answer.

(c) Is it possible for the worker cover each part of the walk exactly once if he doesn't need to end up where he started? (That is, does the graph have an Eulerian path?) Justify your answer.

4. A person starting in Wichita must visit Kansas City, Omaha, and St. Louis (in any order), then return home to Wichita. You don't need to know that Omaha is north of Wichita, Kansas City is northeast, and St. Louis is east, but I'll tell you anyway. Approximate road mileages between the various cities are given below.

	Kansas City	**Omaha**	**St. Louis**	**Wichita**
Kansas City	—	220	225	280
Omaha	220	—	310	300
St. Louis	225	310	—	500
Wichita	280	300	500	—

(a) Draw a weighted graph which corresponds to the situation.

(b) Use the nearest neighbor algorithm to find an approximate solution to the traveling salesman problem, making a circuit starting at Wichita. What is the length of this circuit? (Write your answers in the spaces below.

Wichita, _____, _____, _____, Wichita. Distance: _____

(c) Use the cheapest link algorithm to find an approximate solution to the traveling salesman problem, making a circuit starting at Wichita. What is the length of this circuit?(Write your answers in the spaces below.

Wichita, _____, _____, _____, Wichita. Distance: _____

(d) Write the itinerary of the *shortest possible* solution to the traveling salesman problem in ths case.

Wichita, _____, _____, _____, Wichita. Distance: _____