Quiz 2

June 16 2003

Name:

(1) For the following differential equation use the improved Euler method to estimate $y(1)$ with two steps.

$$\frac{dy}{dx} = -2xy, \quad y(0) = 2.$$

soloution:

(There was an error in the first version of the solution which was corrected thanks to a comment by Kenneth Silkit Kwan.)

$$f(x,y) = -2xy, \quad x_0 = 0, \quad y_0 = 2, \quad h = 0.5$$

$$u_1 = y_0 + f(x_0, y_0)h = 2 + 0 \times 0.5 = 2,$$

$$y_1 = y_0 + \frac{1}{2}(f(x_0,y_0) + f(x_1,u_1))h = 2 + \frac{1}{2}(0 - 2\times 0.5 \times 2)(0.5) = 2 - 0.5 = 1.5$$

$$u_2 = y_1 + f(x_1, y_1)h = 1.5 - 1.5 \times 0.5 = 0.75$$

$$y_2 = y_1 + \frac{1}{2}(f(x_1,y_1) + f(x_2,u_2))h = 1.5 + \frac{1}{2}(-1.5 - 2\times 1 \times 0.75)(0.5) = 1.5 - 0.75 = 0.75$$

$$\Rightarrow y(1) \simeq 0.75$$

(2) Consider the differential equation $\frac{dx}{dt} = x^2 - 5x + 4$. Analyze $x^2 - 5x + 4$ and use this to find stable and unstable equilibrium solutions for the equation and sketch typical solution curves of the equation.

(3) Suppose that a motorboat is moving at $40ft/s$, when its motor suddenly quits and that after $10s$ the boat has slowed to $20ft/s$. Assume that the resistance while coasting is proportional to its velocity. How far will the boat coast in all?

solution:

$$v(0) = 40, \quad v(10) = 20, \quad , m\frac{dv}{dt} = -kv$$

When k is a constant. If we consider $\rho = k/m$ then:

$$\frac{dv}{dt} = -\rho v$$
$$\Rightarrow \frac{dv}{v} = -\rho dt$$
$$\Rightarrow \int \frac{dv}{v} = -\int \rho dt$$
$$\Rightarrow \ln v = -\rho t + C$$
$$v(0) = 40 \Rightarrow C = \ln 40$$
$$v(10) = 20 \Rightarrow \ln 40 - 10\rho = \ln 20 \Rightarrow \rho = \frac{\ln 2}{10}$$
$$\Rightarrow v(t) = 40e^{-\frac{\ln 2}{10}t}$$

But all the distance that it travels is:

$$\int_0^\infty v(t)dt = \int_0^\infty 40e^{-\frac{\ln 2}{10}t}dt$$
$$= (-\frac{400}{\ln 2}e^{-\frac{\ln 2}{10}t}]_0^\infty = \frac{400}{\ln 2}$$