Problem Set I

Due Feb. 5th

Think about all the problems and try to come up with ideas to solve them and write those. Write a complete solution for at least two of the problems. The solutions have to be clear and convincing for a skeptical classmate.

1. Beginning with 2 and 7 the sequence 2, 7, 1, 4, 7, 4, 2, 8, ... is constructed by multiplying successive pairs of it members and adjoining the result as the next one or two members of the sequence, depending on whether the product is a one- or a two-digit number. Prove that the digit 6 appears an infinite number of times in the sequence.

2. We have a convex $2n$-gon (a polygon with $2n$ vertices). Is it possible to choose a point inside and connect it to all the vertices and continue, such that each obtained line intersects exactly one of the sides of the polygon?

3. On planet Markar there are 132 countries. Prove that at least two of them have the same number of neighbors.

4. At the vertices of a regular hexagon are written six nonnegative integers whose sum is 2003. Bert is allowed to make moves of the following form: he may pick a vertex and replace the number written there by the absolute value of the difference between the numbers written at the two neighboring vertices. Prove that Bert can make a sequence of moves, after which the number 0 appears at all six vertices.

5. Find a general formula for the nth derivative of $f(x) = 1/(1 - x^2)$.

6. We have a 4×5 Chocolate bar and we want to break it down into units (1×1 pieces) of chocolate. To do so, each time we can break a piece of chocolate along one of the straight lines on it into two pieces. We cannot break two pieces at a time. What is the smallest number of breaks needed for completing the process?