Homework VI

Elementary Logic
Due Oct. 28th

As usual, think about all problems, solve at least two of the problems. Your answers have to be justified and clear.

A number Machine. Consider the *Curious Number Machine*, we described in class. We can put a number into the machine and after a while a number comes out of the machine. By a *number*, we mean a positive integer considered as a string of digits without any 0 occuring in it, i.e. a string of digits 1,2,3,4,5,6,7,8 and 9. Given two numbers N and M, by NM we don't mean N times M, but the number obtained by first writing the digits of N in the order in which they occur, and then following it by digits of M. For example, if N is 53 and M is 719, by NM we mean 53719.
The machine accepts only some numbers, which we call *acceptable*. We say X *produces* Y, if X is acceptable and that when X is put into the machine, Y is the number that comes out.

- **Rule 1.** For any number X, the number $2X$ is acceptable and produces X.

As in class, we call the number $X2X$ the *associate* of the number X. For example, the associate of 7 is 727 and the associate of 792 is 7922792.

- **Rule 2.** For any numbers X and Y, if X produces Y, then $3X$ produces the associate of Y, i.e. $Y2Y$.

We have two more rules here. First note that by *reverse* of a number we mean the number written backwards; for example, the reverse of 7129 is 9217. Also, for any number X, we refer to XX as the *repeat* of X; for example, 78927892 is the repeat of 7892.

- **Rule 3.** For any numbers X and Y, if X produces Y, then $4X$ produces the reverse of Y.
- **Rule 4.** If X produces Y, then $5X$ produces YY, the repeat of Y.

1. Consider the machine described above and try doing each of these:
   (a) Find a number N, which produces $7N$.
   (b) Find a *non-symmetric* number X, one which is not the same as its inverse, and it produces its own inverse.
   (c) Find a number N, which produces the repeat of N.
2. Now you may want to try these:
   (a) Can you find X which produces the reverse of repeat of X?

(b) Try to generalize these and come up with a general formula for a number X, that produces a transform of X. The transform can be anything like, the associate, the repeat of inverse or the repeat of associate of repeat of inverse; we like to call these transformations respectively 3, 54 or 5354. (Do you see why?) Try that for some examples.

Knights and Knaves. We talked in class about an island on which every inhabitant is either a knight or a knave; knights always tell the truth and knaves always lie. As a warmup example, try to see if you can find whether your way is on your right or left just by asking one question from a passing islander.

3. You are introduced to three people on the island, Alicia, Bernice and Carl. You know that one is a knight, one is a knave and one is a visitor who sometimes lies and sometimes tells the truth. During your conversation, Alicia tells you that she is a visitor. Bernice tells you that Alicia and Carl sometimes tell the truth and Carl tells you that Bernice is a visitor. Who is who?

4. A logician once visited this island and came across two inhabitants, A and B. He asked A, "Are both of you knights?" A answered either yes or no. The logician thought for a while bud did not yet have enough information to determine what they were. The logician then asked A, "Are you tow of the same type?" (Same type means both knights or both knaves.) A answered either yes or no, and the logician then knew what type each one was. What type is each?

5. Coming now to modern times, a logic student was once on a date and said to the young lady: "I'd like to ask you a little favor. I will make a statement. All I ask that if the statement is true you give me a photograph of yourself. Will you do that for me?" The lady assented. "But also," continued the man, " if my statement is false, then I want you to promise *not* to give me your photograph. Agreed?" The lady agreed.

   The man then cleverly made a statement such that the lady, after thinking of it for a while, realized (to her secret amusement) that in order to keep her word, she would have to give him not her photograph but a kiss!

   What statement would work?