Homework V

Elementary Counting

Due Oct. 21st

As usual, think about all problems, and come up with some ideas about how to solve them. If you want you can write them. Then choose two of the problems and write up a careful solution. Your solution should be convincing for a skeptical classmate.

For the following problems you need to know these facts:

- The number of permutations of n different objects is

  $$n! = n(n-1)(n-2)\cdots 2.1$$

  with the convention $0! = 1$.

  Example: There are $10! = 3628800$ different ways to seat 10 students in a row.

- If we have n_1 identical objects of type 1, n_2 identical objects of type 2, ... and finally n_k identical objects of type k, the number of permutations of all these $n_1 + n_2 + \cdots + n_k$ objects is

  $$\frac{(n_1 + n_2 + \cdots + n_k)!}{n_1! n_2! \cdots n_k!}.$$

  Example: Having 2 red, 3 green and 5 blue balls, there are $(2+3+5)!/(2!3!5!) = 2520$ different ways to arrange them in a row.

1. As we asked in class, what will be the number of ways for n students to sit around a round table this time. Note that different ways obtained by rotating them around the table will be considered the same.

2. On planet Markar, a social security number is a 5 digit number when each digit is on of the numbers 0,1,2,3,4 or 5. How many different S.S. numbers they can have if all the numbers can be in any of the places?

   What if 0 is not allowed to be the first digit?

   What if in addition to above, they cannot have more than two 1s in one number?

3. How many 10 digit numbers have at least 2 digits equal?

4. For each of the following examples, find the number of different words you can obtain by permuting the letters. (A word doesn't have to make sense.)

   "CLOSENESS" "VECTOR" "TRUST" "CARAVAN"

5. Simplify the following expressions:

   $$n!(n+1) \qquad \frac{n!}{(n-1)!} \qquad \frac{(n+1)!}{n(n+1)}$$

6. On a 5×8 board, a rook is placed on the lower left corner of the board. Assume that the rook can only move horizontally to right or vertically upward. In how many different ways, it can reach the upper right corner of the board? (*Hint:* Every possible path corresponds to a word having eight R's, when it is moving a unit rightward, and five U's, when it is moving a unit upward, and every such word determines a path. For example, the word RUURRRURRUURR corresponds to the path determined by moving a unit right, two units up, three units right, a unit up, two units right, two units up and finally two units right to get to the upper right corner.)