

**Homework II**  
Mathematical Induction  
*Due Sep. 30*

For most of the following problems you can use Mathematical Induction. You have to show all the steps of induction very carefully and write them down. Again you are required to solve two problems completely and write them down. But you are encouraged to think about all problems and if you want write your ideas about those.

1. Prove that  $1 + 3 + 5 + \dots + (2n - 1) = n^2$  for any  $n \geq 1$ .
2. Guess a formula for the sum  $1 \times 2 + 2 \times 3 + \dots + n(n + 1)$  and prove it by induction.
3. Prove that for any  $n \geq 1$ ,  $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} < 2\sqrt{n}$ .
4. Use induction to prove that a set of  $n$  elements has  $2^n$  subsets.
5. In a tournament  $n$  teams participate. Each two teams play exactly once and every game ends by a team winning and the other losing. Show that independent of the results, at the end, we can always put the teams in a row  $P_1, P_2, \dots, P_n$ , such that  $P_1$  has won  $P_2$ ,  $P_2$  has won  $P_3$ ,  $P_3$  has won  $P_4$  and so on.
6. The plane is cut into regions by a finite number of straight lines. Prove that one can color the regions either Red or Blue so that two adjacent regions have different colors. Here adjacent means that they share a common boundary segment. (*Hint*: Use induction on number of lines.)