# Homework XII

Elementary Geometry
*Due Dec. 9th*

As usual, think about all problems, you need to solve at least one problem from this set. But take a look at our previous homeworks and do one of the homeworks that you hadn't done or you did but it was not correct. As usual your answers have to be justified and clear.

Recall some elementary facts from elementary geometry and what we discussed in class:

- Among all paths connecting two points $A$ and $B$ in the plane, the straight line segment $AB$ has the smallest possible length. We denote this length by $|AB|$.
- As a collary, we have the so-called *triangle inequality*: For any three points $A$, $B$ and $C$ in the plane,
$$|AB| \leq |AC| + |CB|.$$
- A region $R$ in the plane is called *convex* if for any two points $A$ and $B$ in $R$, the entire line segment $AB$ is contained in $R$. For example, a triangle is always convex.

1. Show that among any five points in the plane, no three of which are collinear, four of them make vertices of a convex quadrilateral.
2. Inside a convex quadrilateral $ABCD$, find a point $O$ such that the sum of lengths $|OA| + |OB| + |OC| + |OD|$ is the smallest.
3. There are a few islands in a 100-meter wide river, whose perimetrs add up to 800 meters. The banks of the river are parallel lines. Gabriel claims that wherever he starts, he is able to cross the river by rowing at most 300 meters. Should one believe this?
4. Prove that in every convex pentagon, the sum of the lengths of all five diagonals is greater than the perimeter but is less than twice the perimeter.
5. Consider a point $O$ on the hypotenuse $BC$ of a right triangle $ABC$. Draw perpendiculars $OE$ and $OF$ to $AB$ and $AC$, respectively. Determine the position of $O$ for which the length $|EF|$ is minimum.