

Homework X

Games

Due Nov. 25th

As usual, think about all problems, solve at least two of the problems. Your answers have to be justified and clear.

In each problem, there is a game in which two players take turns making moves, and a player cannot decline to move. The problem is always to find out which player (if any) has a winning strategy. *Try playing the games first.* Start with simpler cases and try to find a pattern by understanding each simple case completely.

Recall that a *winning strategy* is a strategy that one player can follow to guarantee that s/he will win.

1. Consider the game we discussed in class. Two players start with a pile of 27 stones. In each turn, a player can remove either one or two stones from the pile. The one who removes the last stone will be the winner of the game. Who has a strategy for winning and what is that?
2. A rook stands at the bottom left corner of a chessboard. Players take turns moving the rook as many squares as they want, but it can be either horizontally to the right or vertically upward. The player who places the rook on the top right corner square wins. Who can win and how?
3. Same as the last game but with a king and a king's moves. It starts on the bottom left corner again. Each time a player can move it either one square to the right, or one square upward, or diagonally one upward and one to the right. Again the player who places the king on the top right corner square wins. Who can win and how?
4. There is a pile of 33 pennies on a table. The first player divides the pile into two smaller piles (not necessarily equal). The second player chooses a pile and divides it into two smaller piles. They continue. Each turn a player chooses one of the piles on the table and divides it into two smaller piles. The player who cannot do this anymore is the loser. Who wins and how?
5. The same game as what we had in class. There are two piles of stones. One pile has 30 stones and the other has 26 stones. Players take turns removing as many stones as they want (at least one) from *one* pile. This time the player who removes the last stone *loses!* Who can win and how?