Solutions to Practice Final Exam

1. Using the Black Scholes formula, compute the price of a call option with strike price $X = 45$ expiring in 156 days. The current stock price is $S = 44.375$, the riskless rate is $r = 0.07$ and the volatility is $\sigma = 0.31$.

   In this case $T = \frac{156}{356}$. Applying the formula:

$$d_1 = \frac{\log\left(\frac{S}{X}\right) + rT}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\ln\left(\frac{44.375}{45}\right) + 0.07 \times \frac{156}{365}}{0.31 \times \sqrt{\frac{156}{365}}} + \frac{1}{2} \times 0.31 \times \sqrt{\frac{156}{365}} = 0.17994$$

$$d_2 = \frac{\ln\left(\frac{44.375}{45}\right) + 0.07 \times \frac{156}{365}}{0.31 \times \sqrt{\frac{156}{365}}} - \frac{1}{2} \times 0.31 \times \sqrt{\frac{156}{365}} = -0.022722$$

   we obtain

$$N(d_1) = \int_{-\infty}^{0.17994} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx = 0.5714$$

$$N(d_2) = \int_{-\infty}^{-0.022722} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx = 0.49094$$

   and the price of the option is

$$c = 44.375 \times 0.5714 - 45e^{-0.07 \times \frac{156}{365}} \times 0.490\,94 = 3.9147.$$

2. Consider the following combination:

   Buy a put option with strike price $X = 100$.

   Buy a call option with strike price $X = 100$.

   Sell a call option with strike price $X = 200$.

   The final payoff is given by the following table

   which is exactly the desired payoff.

	$S_T < 100$	$100 \leq S_T < 200$	$200 \leq S_T$
put 100	$100 - S_T$	0	0
call 100	0	$S_T - 100$	$S_T - 100$
−call 200	0	0	$-(S_T - 200)$
Total	$100 - S_T$	$S_T - 100$	100

3. We have $r = 0.05$, earnings per period equal to 400000, dividends equal earnings.

   (a) If there are 100000 shares then each share receives a dividend of 4 is each period. The price p_0 of each share is

   $$p_0 = \sum_{t=1}^{\infty} \frac{4}{(1.05)^t} = \frac{4}{0.05} = 80.$$

   (b) The company now announces that it will use the earnings of period 1 to buy back shares.

      i. Let N be the number of shares which are repurchased. The price p_1 has to satisfy

      $$p_1 = \frac{400000}{(100000 - N)\, 0.05}$$

      Furthermore, the amount spent has to be 400000. Therefore p_1 and N must satisfy

      $$Np = 400000$$

      The solution of this system is $N = 4761.9, p_1 = 84$. Therefore, from period 1 on there will be only $100000 - 4761.9 = 95238.1$ shares.

      ii. To compute p_0 after he company decides to change the policy, observe that now the firm does not pay dividends next period. Therefore the current price is given by

      $$p_0 = \frac{p_1}{1 + r}$$

which in our case works out to

$$p_0 = \frac{84}{1.05} = 80.$$

Notice that this conclusion was to be expected, given the Modigliani-Miller theorem on the irrelevance of dividends.