

f) Assume there is a subdivision of S^2 such that each face is a hexagon and any two faces have at most one edge in common.

Since there are six edges per face and each edge belongs to exactly two faces we have

$$6f = 2e$$

$$\Rightarrow 3f = e \quad (1)$$

Since each face has 6 vertices and each vertex belongs to at least three faces we have

$$6f \geq 3v$$

$$\Rightarrow 2f \geq v \quad (2)$$

Finally, because we are considering a subdivision of S^2 we have:

$$v - e + f = 2$$

$$\Rightarrow v - 3f + f = 2 \quad \text{by (1)}$$

$$\Rightarrow 2f - 3f + f \leq 2 \quad \text{by (2)}$$

$$\Rightarrow 0 \leq 2 \quad \text{---}$$