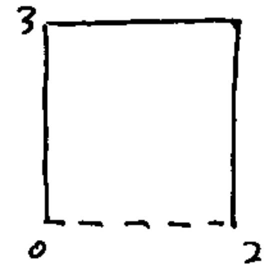
The function $x \mapsto -2x^2+2$ will also do.
(Draw this).

(4) $X = (0,1] \times [0,2] \approx$ 

$Y = [0,2] \times (0,3] \approx$ 

It is clear that the first interval in the description of X is homeomorphic to the second interval of Y.

Also, the second interval of X is homeomorphic to the first interval in the description of Y.

So try the map

$$f : (x,y) \longmapsto (y, 3x)$$

(5) Show that $(0,1) \cup \{1.01\}$ is not connected.

Let $U = (0,1)$ and $V = \{1.01\}$.

The only thing that is not obvious is that V is open in $X = (0,1) \cup \{1.01\}$.

~~But and any ball around 1.01~~

But $U_r(1.01) \cap X = 1.01$ when $r < .01$