

Note that f doesn't change the second coordinate.

$$\text{So, } f(x, y) = f(x', y') \Rightarrow y = y'$$

We just ~~need~~ need to show that $x = x'$.

If y (and hence y') is < 0 then

$$f(x, y) = f(x', y') \Rightarrow (x, y) = (x', y') \text{ and we are done.}$$

If y (and hence y') is ≥ 0 then

$$f(x, y) = f(x', y') \Rightarrow (x+y, y) = (x'+y', y')$$

$$\Rightarrow \begin{cases} x+y = x'+y' \\ y = y' \end{cases}$$

$$\Rightarrow x = x' \text{ and } y = y'$$

- to show that f is onto we need to find for every $(x', y') \in \mathbb{R}^2$ a pt $(x, y) \in \mathbb{R}^2$ such that $f(x, y) = (x', y')$.

If $y' < 0$ then certainly $(x, y) = (x', y')$ will do.

If $y' \geq 0$ then $(x, y) = (x' - y', y')$ satisfies

$$f(x' - y', y') = (x' - y' + y', y') = (x', y').$$