4)

- First we show that $(a,b) \sim (-1,1)$

Consider the map

$$h : (a,b) \to (-1,1)$$
$$t \mapsto \frac{2}{b-a}t - \frac{2a}{b-a} - 1$$

this is clearly continuous and has continuous inverse

$$h^{-1} : (-1,1) \to (a,b)$$
$$t \mapsto \frac{b-a}{2}t + \frac{b-a}{2} + a$$

- Now we show that $(-1,1) \sim X = \{(x_1,x_2) \mid x_1^2+x_2^2=1, x_1>0\}$

Consider the map

$$g : (-1,1) \to X$$
$$x \mapsto (\sqrt{1-x^2}, x)$$

this is continuous and has continuous inverse

$$g^{-1} : X \to (-1,1)$$
$$(x_1,x_2) \mapsto x_2 .$$

- Finally, $g \circ h : (a,b) \to X$ is a homeomorphism.