(6) Consider the map $\phi: [0,1] \to [0, 1/2]$.
$$x \mapsto x/2$$

ϕ is one-to-one so the cardinality of $[0, 1/2]$ is at least "as big" as the cardinality of $[0,1]$.

We proved in class that $[0,1]$ is uncountable, hence $[0,1/2]$ is uncountable.

Alternatively, assume that $[0,1/2]$ is countable.

Then we could count the images of $[0,1]$ under ϕ.

But, since ϕ is one-to-one, this would mean that we could count the elements of $[0,1]$.

This is a contradiction $\not\!\!\!\!\nearrow$