$r < x_2 - a_2$, $r < x_1 - a_1$, and $r < b_1 - x_1$

Overall this can be achieved by choosing

$$r < \min \{ |x_1 - a_1| , |x_1 - b_1| , |x_2 - b_2| , |x_2 - a_2| \}$$

We ~~we~~ need to check that this is small enough.

That is, for $y = (y_1, y_2) \in U_r(x)$ we need to show that if r is as above then y is also in the rectangle.

Now $y \in U_r(x) \Rightarrow d(y,x) < r$

$$\Rightarrow \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2} < r .$$

$$\Rightarrow |y_1 - x_1| \text{ \underline{and} } |y_2 - x_2| \text{ are } < r$$

But $|y_1 - x_1| < r$

$\Leftrightarrow \quad -r < y_1 - x_1 < r$

$\Rightarrow \quad a_1 - x_1 < y_1 - x_1 < b_1 - x_1$ (by our choice of r).

$\Rightarrow \quad a_1 < y_1 < b_1$

In the same way $a_2 < y_2 < b_2$, so y is in the rectangle. So, $U_r(x)$ is inside the rectangle, x is an interior point and we are done.