③ Consider the closed sets $A_n = [0, 1-\frac{1}{n}]$ in $\mathbb{R}^1$.

We claim that $\bigcup_{n=1}^{\infty} A_n = [0,1)$ which is not closed.

To prove that the two sets are equal we must show $\bigcup_{n=1}^{\infty} A_n \subset [0,1)$ and $[0,1) \subset \bigcup_{n=1}^{\infty} A_n$.

- To show that $\bigcup_{n=1}^{\infty} A_n \subset [0,1)$ we must show that every $x \in \bigcup_{n=1}^{\infty} A_n$ is in $[0,1)$.

  Let $x \in \bigcup_{n=1}^{\infty} A_n$. Then x must belong to one of the sets A_n. But each A_n is contained in $[0,1)$ so we have

  $$x \in A_n \text{ for some } n \subset [0,1).$$

- Now we show that $[0,1) \subset \bigcup_{n=1}^{\infty} A_n$.

  Given $x \in [0,1)$ we must show $x \in A_n$ for some n.

  Since $A_n = [0, 1-\frac{1}{n}]$, this is the same as showing that $x \leq 1-\frac{1}{n}$ for some n.