4. Prove that it is impossible to find a subdivision of the sphere such that each face is a hexagon and any two faces have at most one edge in common.

   Hint: Assume such a subdivision exists and check that it must satisfy $e = 3f$ and $v \leq 2f$. Then use the Euler characteristic to get a contradiction.

5. Consider any polyhedron with v vertices, e edges and f faces. Let v_n be the number of vertices at which n edges meet, and let f_n be the number of faces with n edges (n-gons). Show that

$$v_3 + v_4 + \cdots = v$$

$$f_3 + f_4 + \cdots = f$$

   and

$$3v_3 + 4v_4 + \cdots = 3f_3 + 4f_4 + \cdots = 2e.$$

6. The standard soccer ball gives a good example of a polyhedral structure on the sphere, with 12 pentagonal and 20 hexagonal faces. Find v, e and f for this polyhedron and verify that $v - e + f = 2$. Find the v_n's and f_n's and verify the relations in the previous problem.

7. For each vector field, find the index of the singular point at the center of the picture