Math 364 Problem Sheet 1: Due Sept. 16

1. Let's say that two lower-case letters of the English alphabet are *equivalent* if one can be continuously deformed into the other (think as if they are made of rubber which you can bend and stretch but can not break). For example, **a** is equivalent to **b** and to **d** but is not equivalent to **h**. Make a list of all equivalent lower-case letters.

2. Let x, y be points in $\mathbb{R}^n$. The *closed line segment from x to y* is the set

$$[x, y] = \{tx + (1-t)y \mid 0 \leq t \leq 1\}.$$

   A subset $A \subset \mathbb{R}^n$ is said to be *convex* if for every $x, y \in A$ the closed line segment $[x, y]$ is contained in A.

   Prove that open balls $U_r(x_0)$ and closed balls $B_r(x_0)$ are both convex.

3. Give an example of a countably infinite family $A_1, A_2, \ldots$ of closed sets such that $\cup_{j=1}^{\infty} A_j$ is not closed.

4. If $a_j < b_j$ for each $j = 1, 2, \ldots n$, then the subset $(a_1, b_1) \times (a_2, b_2) \times \cdots \times (a_n, b_n)$ of $\mathbb{R}^n$ is called an *open rectangle* in $\mathbb{R}^n$. The subset $[a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_n, b_n]$ of $\mathbb{R}^n$ is called a *closed rectangle* in $\mathbb{R}^n$.

   Show that an open rectangle is an open subset and a closed rectangle is a closed subset of $\mathbb{R}^n$.

5. Show that a subset U of $\mathbb{R}^n$ is open if and only if, for each $x_0 \in U$, there is an open rectangle in $\mathbb{R}^n$ containing x_0 and contained in U.