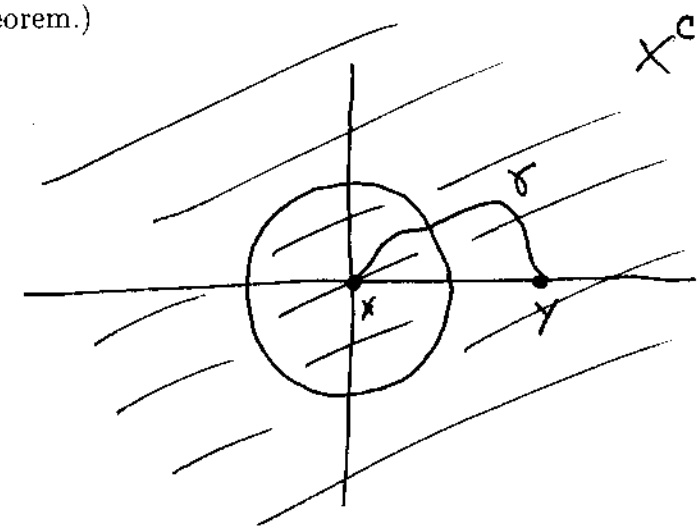
4. (5 points) Let $X \subset \mathbb{R}^2$ be the set $X = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$. Prove that **the complement of** X **is not** path-connected. (Hint: use the Intermediate Value Theorem.)



Let $x = (0,0)$ and $y = (2,0)$. We will show that every path $\gamma : [0,1] \to \mathbb{R}^2$ from x to y must pass through X.

The key to proving this is to note that γ crosses the set X iff there is some ~~there must be some~~ $t \in [0,1]$ such that $\|\gamma(t)\|^2 = 1$.

To prove that such a $t \in [0,1]$ exists we apply the Intermediate Value Theorem to the function $t \to \|\gamma(t)\|^2$.

This function is continuous, its value at $t=0$ is $\|(0,0)\|^2 = 0$ and it's value at $t=1$ is $\|(2,0)\|^2 = 4$.
So, by the IVT there is a $t \in [0,1]$ such that $\|\gamma(t)\|^2 = 1$.