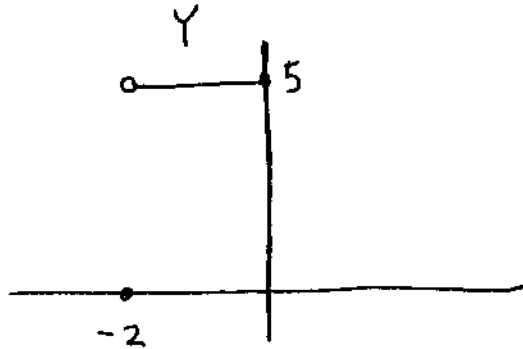
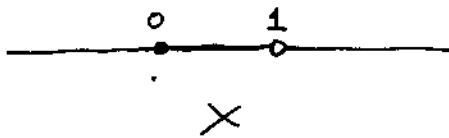


3. (5 points) Write down a formula for a homeomorphism from  $X = [0, 1) \subset \mathbb{R}^1$  to  $Y = \{(x, y) \in \mathbb{R}^2 \mid x \in [-2, 0] \text{ and } y = 5\}$ . If you prefer you can write the map as a composition of simpler homeomorphisms. (Remember to show that your map has a continuous inverse.)



The map  $x \mapsto -2x$  is a homeomorphism from  $[0, 1) \subset \mathbb{R}^1$  to  $(-2, 0] \subset \mathbb{R}^1$ . To turn this into the homeomorphism we want we just need to put the image in  $\mathbb{R}^2$  at a height of 5.

So, the map should be  $h : X \rightarrow Y$   
 $x \mapsto (-2x, 5)$

The map  $h$  is continuous because both of its ~~could~~ components are continuous.

The inverse of  $h$  is the map  $h^{-1} : Y \rightarrow X$   
 $(x_1, x_2) \mapsto -\frac{1}{2}x_1$

This is also continuous so  $h : X \rightarrow Y$  is a homeomorphism.

Z