SOLUTIONS TO THE PRACTICE FINAL

(1) $\sin(-2\pi) = \sin(0) = 0$. This is because -2π and 0 are equivalent angles. So the answer is 'c'.

(2) First calculate $\cos(\pi/4) = \sqrt{2}/2$. Remember that to find $\arccos(\sqrt{2}/2)$ we need to find an angle θ such that
- $0 \leq \theta \leq \pi$
- and $\cos(\theta) = \sqrt{2}/2$.

The solution is $\theta = \pi/4$. Notice that in this case $\arccos(\cos(\theta)) = \theta$, but this is not always the case. This only happened because $0 \leq \pi/4 \leq \pi$. So the answer is 'b'.

(3) Calculate $\arcsin(1) = \pi/2$ since $\sin(\pi/2) = 1$ and $-\pi/2 \leq \pi/2 \leq \pi/2$. Also calculate $\arccos(0) = \pi/2$ since $\cos(\pi/2) = 0$ and $0 \leq \pi/2 \leq \pi$. So, the answer is 'a'.

(4) Draw a right triangle with an angle marked θ whose opposite is $o = 3$, whose adjacent a is unknown and the hypotenuse is $h = 5$. Then $\sin(\theta) = o/h = 3/5$ as required. By the Pythagorean theorem, $a^2 + o^2 = h^2$. So $a^2 + 3^2 = 5^2$ and solving for a, get $a = \pm 4$. In order for $\cos(\theta) = a/h$ to be negative we need to choose $a = -4$. Then $\cos(\theta) = -4/5$ and $\sec(\theta) = 1/\cos(\theta) = -5/4$. So the answer is 'a'.

(5) On the unit circle there is only one angle x with $\sin(x) = -1$, that is $\sin(3\pi/2) = -1$. (This is a special case since the line $y = -1$ intersects the unit circle at a single point. In general it may intersect at 0 or 2 points.) Any equivalent angle will also be a solution so the general solution is $3\pi/2 + n(2\pi)$, where n is any integer. So the answer is 'd'.

(6) $g(x)$ has 2 vertical asymptotes at each of the zeros $x = \pm 4$ of the denominator $x^2 - 16$. So the answer is 'b'.

(7) Recall that zeros of the numerator are zeros of the function $g(x)$, that is points x where $y = g(x) = 0$. Thus, they are x-intercepts of $g(x)$. So the answer is 'a'.

(8) The ratio of the leading coefficients is $\frac{1}{1} = 1$ so there is a horizontal asymptote at $y = 1$. This means that as $x \to \infty$, $g(x) \to 1$. So the answer is 'b'.

(9) $\frac{x^2+4}{x-3}$ has no zeros since $x^2 + 4$ has no zeros since $x^2 \geq 0 \Rightarrow x^2 + 4 \geq 4 > 0$. Thus $\frac{x^2+4}{x-3}$ has no x-intercept. So the answer is 'c'.

(10) Since the sign of $f'(t)$ is negative for t in the interval $(0, 2)$, f is decreasing in this interval; that is the price of Tbay stock fell during the first 2 hours. The second derivative is the rate at which the derivative is changing. Since $f''(t)$ is negative in the given interval, f' is decreasing in this interval; that is the rate of change of the price of Tbay stock is decreasing. Careful: this means price of Tbay stock falls faster as time progresses. Think about the graph of $-x^2$ when $x > 0$ if you want to visualize this. So the answer is 'd'.

(11) At $t = 2$ the second derivative goes from being negative to being positive. The problem notes that the price increased only shortly after the announcement. At $t = 2.1$ the first derivative goes from being negative to positive. We can interpret this as meaning that the announcement came at $t = 2$ when the price started 'accelerating' upward and shortly later at $t = 2.1$, it started rising. 2 hours after $9:30$am is $11:30$am. So the answer is 'b'.
(12) Graph $y = \sin(x)$ and notice that it is increasing for $0 < x < \pi/2$. Thus the derivative is positive. So the answer is 'a'.
(13) The graph has a sharp point at $x = 3$ and hence the function cannot be differentiable so I is false. The function is increasing for $x < 3$ so II is true. The slope is the same at $x = 0$ and at $x = 1$ so $f'(0) = f'(1)$ and III is false. So the answer is 'b'.
(14) One of the edges is already labeled x; label the 3 parallel edges also with x. Label the top and bottom edge with y. The area of the yard is then $A = x \cdot y$ but the question asks for area as a 'function of x' so we must first find y as a function of x. There are 4 edges labeled x and 2 edges labeled y so the total fencing is $4x + 2y = 800$. Solving for y, $y = 400 - 2x$. Plugging in, $A = x(400 - 2x)$. So the answer is 'd'.
(15) Be careful! This is a graph of $f''(x)$, the second derivative. We know that it is positive if $f(x)$ is concave up and negative if $f(x)$ is concave down. Equivalently it is positive if $f'(x)$ is increasing and negative if $f'(x)$ is decreasing. Well, it is positive when $x > 2$ so $f'(x)$ is increasing then. So the answer is 'b'.
(16) If $f(x) = \tan(x)$ then by the definition of derivative as a limit of a difference quotient $f'(x) = \lim_{h\to 0} \frac{\tan(x+h)-\tan(x)}{h}$. So the answer is 'c'.
(17) The slope of the secant line _is_ the difference quotient $\frac{F(x_1)-F(x_0)}{x_1-x_0}$, the rise over the run. Thus the slope is $\frac{25-10}{8-5} = \frac{15}{3} = 5$. So the answer is 'a'.
(18) Draw small tangent lines at each of the given x_i's. Notice that they are flat at both x_1 and x_3. In increasing order of slopes m_i they go $m_4 < m_1 = m_3 < m_2 < m_5$. These slopes are exactly the derivatives $m_i = f'(x_i)$. So the answer is 'b'.
(19) Careful: we're looking at the _second_ derivative. The second derivative is 0 if say it goes from being negative to being positive (or vice versa). Notice that the graph is concave down to the left of x_4 and concave up to the right of x_4 and it goes from being concave up to the left of x_2 to being concave down to the right of x_2. Thus, the second derivative goes from being negative to being positive at x_4 and vice versa at x_2. Thus, $f''(x_4) = f''(x_2) = 0$. x_4 is not given as an answer. So the answer is 'b'.
(20) The slope of the tangent line is $g'(2) = 4$. It is also the rise over the run $\frac{y-1}{3-2} = y-1$. Setting them equal and solving for y, get $y = 5$. So the answer is 'a'.
(21) Notice that the graph of $f(x)$ is increasing to the left of -1 and to the right of 3 and is decreasing between -1 and 3. Thus $f'(x)$ is positive to the left of -1 and to the right of 3 and negative between -1 and 3. So the answer is 'b'.
(22) Let's calculate the derivative using the limit of a difference quotient.

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(1+h)^2 + 2 - 3}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2h}{h} = \lim_{h \to 0} h + 2 = 0 + 2 = 2$$

Alternatively use the power rule and linearity to get $f'(x) = 2x + 0 = 2x$, so $f'(1) = 2$. Thus, the slope of the tangent at $x = 1$ is 2. So the answer is 'b'.

(23) It appeeeears that $f(x)$ is increasing between $x = 1$ and 5. Also the rate of increase appeeeears to be increasing. Thus, apparently $f'(x) > 0$ and $f''(x) > 0$. So the answer is 'a'.

(24) From 23, it appears that the graph of $f(x)$ should increase and be concave up. So the answer is 'a'.

(25) $420°$ is equivalent to $420° - 360° = 60°$ and $\cos(60°) = 1/2$. So the answer is 'a'.

(26) Using the product rule and power rule for logs,

$$1 + \ln(p\sqrt{q}) = 1 + \ln(p) + \ln(\sqrt{q})$$

$$= 1 + \ln(p) + \ln(q^{1/2}) = 1 + \ln(p) + \frac{1}{2}\ln(q)$$

$$= 1 + 3 + \frac{1}{2} \cdot 8 = 8$$

So the answer is 'c'.

(27) Tricky: $g'(2)$ is the same as the slope of l which is $\frac{k-3}{6-2}$ from the graph $k > 5$ so $k - 3 > 2$ so $\frac{k-3}{6-2} > \frac{1}{2}$. Thus the only possible answer among the given ones is 2. So the answer is 'b'.

(28) The graph of $g(x)$ is increasing and concave down. Thus $g'(x) > 0$ and $g''(x) < 0$. Thus the velocity is positive and the acceleration is negative. So the answer is 'b'.

(29) Since $3^3 = 27$, $\log_3 27 = 3$. Thus $2^{\log_3 27} = 2^3 = 8$. So the answer 'b'.

(30) The amplitude is 2 and the period is $2\pi/3$. Other than that it looks like the graph of sin. Thus, the equation is $y = 2\sin(3x)$. So the answer is 'd'.

(31) We didn't discuss the derivatives of exponentials or logarithms, but looking at the graphs we see that $f(x) = e^x$ is both increasing and concave up. Indeed $f'(x) = e^x$ so $f''(x) = e^x$ and $e^x > 0$. So the answer is 'c'.

(32) Since e^x is well-defined for any value of x so is $e^{-x} + 2$. Thus the domain is $(-\infty, \infty)$. So the answer is 'a'.

(33) We need to find θ with $\sin\theta = -1/2$ and $-\pi/2 \le \theta \le \pi/2$. Since $\sin(\pi/6) = 1/2$, $\sin(-\pi/6) = -1/2$. And, $-\pi/2 \le -\pi/6 \le \pi/2$. So the answer is 'd'.

(34) Using the product rule for logarithms $\log_2 x + \log_2(x+14) = \log_2(x(x+14))$. Thus the equality becomes $\log_2(x(x+14)) = \log_2(32)$. This is true if and only if $x(x+14) = 32$. A little algebra and this is equivalent to $x^2 + 14x - 32 = 0$ or $(x-2)(x+16) = 0$. Thus the solution set is $x \in \{2, -16\}$. So the answer is 'b'.

(35) $\ln(x)$ is only well-defined for $x > 0$ so $ln(x-2) - 1$ is only well defined for $x - 2 > 0$ or equivalently $x > 2$, that is $x \in (2, \infty)$. So the answer is 'b'.

(36) Starting with $e^{x-2} = 3$, take natural log of both sides to get $x - 2 = \ln(3)$. Add 2 to both sides to get $x = \ln(3) + 2$. So the answer is 'a'.

(37) We can determine the hypotenuse using Pythagoras' theorem, $\sqrt{(-3)^2 + 4^2} = 5$. Secant is given by hypotenuse over adjacent which is the x-value. Thus $\sec\theta = -\frac{5}{3}$. So the answer is 'b'.

(38) x-intercepts are zeros so we must find where $f(x) = 0$. Thus either $e^{-x} = 0$ or $ln(x) = 0$. But $e^{-x} > 0$ for all x and $ln(x) = 0$ only when $x = 1$. So the solution set is $\{1\}$.

(39) Recall that $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$. This matches the given expression when $f(x) = x^3 + x^2 - x$. So the answer is 'a'.

(40) This looks like the graph of $\ln x$ shifted to the left 2 units. Thus $g(x) = \ln(x + 2)$. So the answer is 'd'.