(1) Perform the following matrix multiplications:

(a) $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 7 \\ 5 & 8 \\ 6 & 9 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 3i \\ 2i & 1 \end{pmatrix} \begin{pmatrix} 1+i & 2+3i \\ 4+2i & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}^T \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

(2) In $\mathbb{R}^4$, are the vectors $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} \right\}$ linearly dependent or linearly independent?

(3) What is a basis for *span* $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} \right\}$?

(4) Is multiplication by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$ injective? Is it surjective?

(5) Using the Gram-Schmidt process, find an orthonormal basis for $\mathbb{R}^3$ starting with the basis $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \right\}$

(6) Find the inverse of the matrix $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.

(7) Find the determinant of:

(a) $a \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 9 & 2 \\ 7 & 8 & 15 & 1 \\ 1 & 2 & 3 & 2 \end{pmatrix}$

(8) Find the eigenvalues of the matrix $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$.

(9) Let A be a square matrix with real entries. Prove that A is invertible with $A^T = A^{-1}$ if and only if the columns of A form an orthonormal basis.

(10) Prove that the set of even polynomials, $\{a_0 + a_2x^2 + \cdots + a_{2n}x^{2n}\}$ is a subspace of the space of polynomials.