HOMEWORK 4

(1) Use row reduction to solve the system of linear equations

$$x + y + z + w = 10$$
$$x - z - 2w = -10$$
$$2x + 3y + z = 11$$
$$4x + 3y + 2z + w = 20$$

(2) Use row reduction to find the inverse of the following matrices

(a)
$$\begin{pmatrix} -1 & -1 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(c)
$$\begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$$

(3) Use the method of least squares to find an equation for a parabola which best fits the data points $(-1, 1), (0, 0), (0, 1)(2, 1)$. Draw a graph of your results.

(4) Consider the following formula for the determinant of a 3×3-matrix

$$det \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} = x_1y_2z_3 + y_1z_2x_3 + z_1x_2y_3 - (x_3y_2z_1 + y_3z_2x_1 + z_3x_2y_1)$$

(a) Show that it is linear in the first column.

(b) Show that it is alternating when you switch the first and second column.

(c) Show that $det \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1$.

(d) Calculate $det \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$.

(5) Use the formula for the determinant of a 3×3-matrix from problem 4, to find the eigenvalues of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$.