

The normal distribution is given by $f(x; \mu, \sigma) = \frac{1}{(2\pi)^{1/2}\sigma} \exp(-\frac{1}{2}(\frac{x-\mu}{\sigma})^2)$ for $-\infty < x < \infty$.

Proof that the normal distribution has $\int_{-\infty}^{\infty} f(x; \mu, \sigma)dx = 1$: Let $y = (x - \mu)/\sigma$. Then $\int_{-\infty}^{\infty} f(x; \mu, \sigma)dx = \int_{-\infty}^{\infty} \frac{1}{(2\pi)^{1/2}} \exp(-\frac{1}{2}y^2)dy$. Therefore, it suffices to show that $I = \int_{-\infty}^{\infty} \exp(-\frac{1}{2}y^2)dy = (2\pi)^{1/2}$. But $I^2 = \int_{-\infty}^{\infty} \exp(-\frac{1}{2}y^2)dy \int_{-\infty}^{\infty} \exp(-\frac{1}{2}z^2)dz = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(-\frac{1}{2}(y^2 + z^2))dydz = \int_0^{2\pi} \int_0^{\infty} \exp(-\frac{1}{2}(r^2))rdrd\theta = 2\pi$.

The Central Limit Theorem: Let X_1, X_2, \dots be a sequence of independent random variables with the same distribution which has expected value μ and standard deviation σ . Let $Z_n = \frac{X_1 + \dots + X_n - n\mu}{\sigma\sqrt{n}}$. Then Z_n "approaches" a standard normal distribution. I.e. $\lim_{n \rightarrow \infty} P(Z_n \leq z) = \Phi(z)$ where Φ is the CDF of a standard normal distribution.

For a function f with period 2π , we define the n th Fourier coefficient to be $F_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \exp(-inx)dx$ and the Fourier series to be $\sum_{n=-\infty}^{\infty} F_n \exp(inx)$. If f is real we can use Euler's identity $\exp(inx) = \cos(nx) + i \sin(nx)$ to derive the Fourier series $\frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$, where $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)dx$ and $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx)dx$. The coefficients a_n, b_n, F_n are related by $F_n = (a_n - ib_n)/2$ and $F_0 = a_0/2$.

For a series x_0, \dots, x_{n-1} the Discrete Fourier Transform gives a sequence f_0, \dots, f_{n-1} according to the formula $f_j = \sum_{k=0}^{n-1} x_k \exp(-2\pi ijk/n)$.