A weak 2-category consists of

- a set of 0-cells
- for any two 0-cells A, B a category $B \leftarrow A$, i.e. a set of 1-cells and for any 1-cells $B \xleftarrow{f,f'} A$, a set $\begin{tikzcd} B & A \arrow[l, bend right, "f"'] \arrow[l, bend left, "f'"] \arrow[l, phantom, "\Downarrow"] \end{tikzcd}$ of 2-cells and for any 1-cell $B \xleftarrow{f} A$ an identity $\begin{tikzcd} B & A \arrow[l, bend right, "f"'] \arrow[l, bend left, "f"] \arrow[l, phantom, "\Downarrow 1_f"] \end{tikzcd}$ and for any 1-cells $\begin{tikzcd}[column sep=large] B & A \arrow[l, bend right=60, "f"'] \arrow[l, "f'" description] \arrow[l, bend left=60, "f''"] \arrow[l, phantom, shift right=3ex, "\Downarrow F"] \arrow[l, phantom, shift left=3ex, "\Downarrow F'"] \end{tikzcd}$ a vertical composite $\begin{tikzcd} B & A \arrow[l, bend right, "f"'] \arrow[l, bend left, "f''"] \arrow[l, phantom, "\Downarrow {\begin{smallmatrix}F\\F'\end{smallmatrix}}"] \end{tikzcd}$ such that:
  - for any 2-cell $\begin{tikzcd} B & A \arrow[l, bend right, "f"'] \arrow[l, bend left, "f'"] \arrow[l, phantom, "\Downarrow F"] \end{tikzcd}$, $\begin{matrix}1_f\\F\end{matrix} = F = \begin{matrix}F\\1_{f'}\end{matrix}$;
  - for any 2-cells $\begin{tikzcd}[column sep=large] B & A \arrow[l, bend right=70, "f"'] \arrow[l, bend right=20, "f'" description] \arrow[l, bend left=20, "f''" description] \arrow[l, bend left=70, "f'''"] \arrow[l, phantom, shift right=4ex, "\Downarrow F"] \arrow[l, phantom, "\Downarrow F'"] \arrow[l, phantom, shift left=4ex, "\Downarrow F''"] \end{tikzcd}$, $\begin{matrix}\underbrace{\overbrace{\begin{matrix}F\\F'\end{matrix}}}\\F''\end{matrix} = \begin{matrix}F\\\underbrace{\overbrace{\begin{matrix}F'\\F''\end{matrix}}}\end{matrix}$.
- An identity, functors from the terminal category to the categories $A \leftarrow A$, i.e. a 1-cell $A \xleftarrow{1_A} A$.
- A horizontal composition, functors from $C \leftarrow B \leftarrow A$ to $C \leftarrow A$, i.e. for any 1-cells $C \xleftarrow{g} B \xleftarrow{f} A$ a 1-cell $C \xleftarrow{gf} A$ and for any 2-cells $\begin{tikzcd} C & B \arrow[l, bend right, "g"'] \arrow[l, bend left, "g'"] \arrow[l, phantom, "\Downarrow G"] & A \arrow[l, bend right, "f"'] \arrow[l, bend left, "f'"] \arrow[l, phantom, "\Downarrow F"] \end{tikzcd}$ a 2-cell $\begin{tikzcd} C & A \arrow[l, bend right, "gf"'] \arrow[l, bend left, "g'f'"] \arrow[l, phantom, "\Downarrow GF"] \end{tikzcd}$ such that $1_g 1_f = 1_{gf}$ and for any

2-cells $\begin{tikzcd} C & B \arrow[l, bend right=60, "g"'] \arrow[l, "g'" description] \arrow[l, bend left=60, "g''"] \arrow[l, phantom, shift right=3, "\Downarrow G"] \arrow[l, phantom, shift left=3, "\Downarrow G'"] & A \arrow[l, bend right=60, "f"'] \arrow[l, "f'" description] \arrow[l, bend left=60, "f''"] \arrow[l, phantom, shift right=3, "\Downarrow F"] \arrow[l, phantom, shift left=3, "\Downarrow F'"] \end{tikzcd}$, $\left\{ \begin{matrix} G \\ G' \end{matrix} \right\} \left\{ \begin{matrix} F \\ F' \end{matrix} \right\} = \begin{matrix} \overbrace{\underbrace{GF}} \\ \overbrace{\underbrace{G'F'}} \end{matrix}$.

- An associator, a natural family of isomorphisms of horizontal composition from $D \leftarrow (C \leftarrow B \leftarrow A)$ to horizontal composition from $(D \leftarrow C \leftarrow B) \leftarrow A$, i.e. for any 1-cells $D \xleftarrow{h} C \xleftarrow{g} B \xleftarrow{f} A$ an invertible 2-cell $\begin{tikzcd} D & A \arrow[l, bend right, "h(gf)"'] \arrow[l, bend left, "(hg)f"] \arrow[l, phantom, "\Downarrow\alpha(h,g,f)"] \end{tikzcd}$ such that for any 2-cells $\begin{tikzcd} D & C \arrow[l, bend right, "h"'] \arrow[l, bend left, "h'"] \arrow[l, phantom, "\Downarrow H"] & B \arrow[l, bend right, "g"'] \arrow[l, bend left, "g'"] \arrow[l, phantom, "\Downarrow G"] & A \arrow[l, bend right, "f"'] \arrow[l, bend left, "f'"] \arrow[l, phantom, "\Downarrow F"] \end{tikzcd}$ the following diagram commutes

$$\begin{tikzcd} (hg)f \arrow[d, Rightarrow, "(HG)F"'] & h(gf) \arrow[l, Rightarrow, "\alpha(h,g,f)"'] \arrow[d, Rightarrow, "H(GF)"] \\ (h'g')f' & h'(g'f') \arrow[l, Rightarrow, "\alpha(h',g',f')"] \end{tikzcd}$$

- A left identifier, a natural family of isomorphisms from horizontal composition with identity on the left to the identity functor on $B \leftarrow A$, i.e. for any 1-cell $B \xleftarrow{f} A$ an invertible 2-cell $f \xLeftarrow{\lambda(f)} 1_B f$ such that for any 2-cell $\begin{tikzcd} B & A \arrow[l, bend right, "f"'] \arrow[l, bend left, "f'"] \arrow[l, phantom, "\Downarrow F"] \end{tikzcd}$ the following diagram commutes

$$\begin{tikzcd} f \arrow[d, Rightarrow, "F"'] & 1_B f \arrow[l, Rightarrow, "\lambda(f)"'] \arrow[d, Rightarrow, "1_{1_B} F"] \\ f' & 1_B f' \arrow[l, Rightarrow, "\lambda(f')"] \end{tikzcd}$$

- A right identifier, a natural family of isomorphisms from the horizontal composition with identity on the right to the identity functor on $B \leftarrow A$, i.e. for any 1-cell $B \xleftarrow{f} A$ an invertible 2-cell $f \xLeftarrow{\rho(f)} f1_A$ such that for any 2-cell $\begin{tikzcd} B \arrow[r, phantom, "\Downarrow F"] & A \arrow[l, bend right, "f"'] \arrow[l, bend left, "f'"] \end{tikzcd}$ the following diagram commutes

$$\begin{tikzcd}
f & f1_A \arrow[l, Rightarrow, "\rho(f)"'] \arrow[d, Rightarrow, "F1_{1_A}"] \\
f' \arrow[from=u, Rightarrow, "F"'] & f'1_A \arrow[l, Rightarrow, "\rho(f')"]
\end{tikzcd}$$

- Furthermore, the following diagrams are required to commute

$$\begin{tikzcd}
 & i((hg)f) \arrow[dl, Rightarrow, "\alpha(i,hg,f)"'] & \\
(i(hg))f \arrow[d, Rightarrow, "\alpha(i,h,g)1_f"'] & & i(h(gf)) \arrow[ul, Rightarrow, "1_i\alpha(h,g,f)"'] \arrow[d, Rightarrow, "\alpha(i,h,gf)"] \\
((ih)g)f & & (ih)(gf) \arrow[ll, Rightarrow, "\alpha(ih,g,f)"]
\end{tikzcd}$$

$$\begin{tikzcd}
(g1_B)f \arrow[dr, Rightarrow, "\rho(g)1_f"'] & & g(1_Bf) \arrow[ll, Rightarrow, "\alpha(g,1_B,f)"'] \arrow[dl, Rightarrow, "1_g\lambda(f)"] \\
 & gf &
\end{tikzcd}$$