Homework/Problems 10

MAT160: Mathematical problems and games. Spring 2005

04/19/2005. Due on 04/24/2005

Instructions: As usual, think about all problems, and come up with some ideas about how to solve them.

1. Local Stony Brook professors Dr. D and Dr. S bump into each other on Stony Brook Rd. They haven't seen each other since Vietnam.

   S: Hey! How have you been?

   D: Great! I got married and I have three daughters now.

   S: Really? How old are they?

   D: Well, the product of their ages is 72, and the sum of their ages is the same as the number on that building over there.

   S: Right, OK ... oh wait ... hmm, I still don't know.

   D: Oh sorry, the oldest one just started to play the piano.

   S: Wonderful! my oldest is the same age!

   How old are the daughters?

2. There is an island of monks where everyone has either brown eyes or red eyes. Monks who have red eyes are cursed, and are supposed to commit suicide at midnight. However, no one ever talks about what color eyes they have, because the monks have a vow of silence. Also, there are no reflective surfaces on the whole island. Thus, no one knows their own eye color; they can only see the eye colors of other people, and not say anything about them. Life goes on, with brown-eyed monks and red-eyed monks living happily together in peace, and no one ever committing suicide. Then one day a tourist visits the island monastery, and, not knowing that he's not supposed to talk about eyes, he states the observation "At least one of you has red eyes." Having acquired this new information, something dramatic happens among the monks.

   What happens?

3. As in the previous problem, analyze in each case what would happen if we assume:

   (a) "There are 10 Brown Eyed Monks"

   (b) "There are at lesat two Red Eyed Monks"

   (c) "There is an odd number of Red Eyed Monks"

   (d) "There is an even number of Red Eyed Monks"

   (e) "There is more than one Red Eyed Monk"

   In each of the next problems, there is a game in which two players take turns making moves, and a player cannot decline to move. The problem is always to find out which player (if any) has a winning strategy. *Try playing the games first.* Start with simpler cases and try to find a pattern by understanding each simple case completely.

Recall that a *winning strategy* is a strategy that one player can follow to guarantee that she/he will win.

4. Two players start with a pile of 27 stones. In each turn, a player can remove one or two stones from the pile. The one who removes the last stone will be the winner of the game. Who has a strategy for winning and what is that?

5. Two players take turns putting pennies on a perfectly round, symmetrical table, one penny per turn, without piling one penny on top of the other. The player who cannot place a penny loses. Who can win and how?

6. There are two piles each containing 7 pennies. At each turn, a player may take as many pennies as she chooses (but at least one) from one of the piles (either pile). A player loses if she cannot move. Who can win and how?

7. Same as previous game, but this time, if a player can't move this time she *wins*.

8. The game starts with number 1 written on the blackboard. The players take turns adding any integer from 1 through 9 to the current number. The player who reaches the number 100 wins. Who can win and how?

9. Consider the previous game, but this time the player who reaches the number 100 (or a bigger number) loses.