

HOMework/PROBLEMS 6

MAT160: Mathematical problems and games.
Spring 2005

03/08/2005. Due on 03/15/2005

Instructions: This problem set consists of only one question: prove that the standard poker hand ranking is correct. This is, compute the probability of each of the ten hands of poker and compare them. You need to use the *counting permutations with repetitions* formula that we used in problem set 4:

If we have n_1 identical objects of type 1, n_2 identical objects of type 2, ... and finally n_k identical objects of type k , the number of permutations of all these $n_1 + n_2 + \dots + n_k$ objects is

$$\frac{(n_1 + n_2 + \dots + n_k)!}{n_1!n_2! \dots n_k!}.$$

Take a moment to think how to compute probabilities by using this formula. WORK IN TEAMS OF THREE PEOPLE, otherwise it will be a lot of work.

Standard Poker Hand Ranking

There are 52 cards in the pack, and the ranking of the individual cards, from high to low, is ace, king, queen, jack, 10, 9, 8, 7, 6, 5, 4, 3, 2. There is no ranking between the suits - so for example the king of hearts and the king of spades are equal.

A poker hand consists of five cards. The categories of hand, from highest to lowest, are listed below. Any hand in a higher category beats any hand in a lower category (so for example any three of a kind beats any two pairs). Between hands in the same category the rank of the individual cards decides which is better, as described in more detail below.

Poker ranks are always based on five cards only.

Some may wonder why I deal with the case of (say) two threes of a kind of equal rank. This obviously cannot arise in standard poker, but such comparisons are needed in some other games using poker combinations - for example poker menteur - and in games with wild cards.

1. ROYAL FLUSH. This is the highest poker hand. It consists of ace, king, queen, jack, ten, all in the same suit. As all suits are equal, all royal flushes are equal.

2. STRAIGHT FLUSH. Five cards of the same suit in sequence - such as ♣J-♣10-♣9-♣8-♣7. Between two straight flushes, the one containing the higher top card is higher. An ace can be counted as low, so ♥5-♥4-♥3-♥2-♥A is a straight flush, but its top card is the five, not the ace, so it is the lowest type of straight flush. The cards cannot "turn the corner": ♦4-♦3-♦2-♦A-♦K is not valid.

3. FOUR OF A KIND. Four cards of the same rank - such as four queens. The fifth card can be anything. This combination is sometimes known as "quads", and in some parts of Europe it is called a "poker", though this term for it is unknown in English. Between two fours of a kind, the one with the higher set of four cards is higher - so 3-3-3-3-A is beaten by 4-4-4-4-2. It can't happen in standard poker, but if in some other game you need to compare two fours of a kind where the sets of four cards are of the same rank, then the one with the higher fifth card is better.

4. FULL HOUSE. This consists of three cards of one rank and two cards of another rank - for example three sevens and two tens (colloquially known as "sevens full" or more specifically "sevens on tens"). When comparing full houses, the rank of the three cards determines which is higher. For example 9-9-9-4-4 beats 8-8-8-A-A. If the threes of a kind were equal, the rank of the pairs would decide.

5. FLUSH. Five cards of the same suit. When comparing two flushes, the highest card determines which is higher. If the highest cards are equal then the second highest card is compared; if those are equal too, then the third highest card, and so on. For example ♠K-♠J-♠9-♠3-♠2 beats ♦K-♦J-♦7-♦6-♦5 because the nine beats the seven.

6. STRAIGHT. Five cards of mixed suits in sequence - for example ♠Q-♦J-♥10-♠9-♣8. When comparing two sequences, the one with the higher ranking top card is better. Ace can count high or low in a straight, but not both at once, so A-K-Q-J-10 and 5-4-3-2-A are valid straights, but 2-A-K-Q-J is not. 5-4-3-2-A is the lowest kind of straight, the top card being the five.

7. THREE OF A KIND. Three cards of the same rank plus two other cards. This combination is also known as Triplets or Trips. When comparing two threes of a kind the hand in which the three equal cards are of higher rank is better. So for example 5-5-5-3-2 beats 4-4-4-K-Q. If you have to compare two threes of a kind where the sets of three are of equal rank, then the higher of the two remaining cards in each hand are compared, and if those are equal, the lower odd card is compared.

8. TWO PAIRS. A pair is two cards of equal rank. In a hand with two pairs, the two pairs are of different ranks (otherwise you would have four of a kind), and there is an odd card to make the hand up to five cards. When comparing hands with two pairs, the hand with the highest pair wins, irrespective of the rank of the other cards - so J-J-2-2-4 beats 10-10-9-9-8 because the jacks beat the tens. If the higher pairs are equal, the lower pairs are compared, so that for example 8-8-6-6-3 beats 8-8-5-5-K. Finally, if both pairs are the same, the odd cards are compared, so Q-Q-5-5-8 beats Q-Q-5-5-4.

9. PAIR. A hand with two cards of equal rank and three other cards which do not match these or each other. When comparing two such hands, the hand with the higher pair

is better - so for example 6-6-4-3-2 beats 5-5-A-K-Q. If the pairs are equal, compare the highest ranking odd cards from each hand; if these are equal compare the second highest odd card, and if these are equal too compare the lowest odd cards. So J-J-A-9-3 beats J-J-A-8-7 because the 9 beats the 8.

10. HIGH CARD. Five cards which do not form any of the combinations listed above. When comparing two such hands, the one with the better highest card wins. If the highest cards are equal the second cards are compared; if they are equal too the third cards are compared, and so on. So A-J-9-5-3 beats A-10-9-6-4 because the jack beats the ten.