

# HOMEWORK/PROBLEMS 5

MAT160: Mathematical problems and games.  
Spring 2005

03/01/2005. Due on 03/08/2005

**Instructions:** Solve the following questions.

**The Pigeonhole Principle:** If  $n + 1$  objects are distributed among  $n$  boxes, one of the boxes will contain at least 2 objects. More generally, if  $kn + 1$  objects ( $k > 0$ ) are distributed among  $n$  boxes, one of the boxes will contain at least  $k + 1$  objects.

1. Prove that for any set of five points in the interior of a square of side length one, there is at least one pair of points whose distance is less than  $\sqrt{2}/2$ .
2. In a chessboard, remove two opposite corners. Is it possible to cover this board with pieces of domino whose size is exactly two board squares?
3. Given any ten integers, show that there is a pair of these integers whose difference is divisible by 9.
4. Fifty-one trees are planted inside a square field with sides of length 100 feet. Show that some set of 3 of these trees must be contained in a square with sides of length 20 feet.
5. A certain dorm has 25 people in it.
  - (a) You are told that each person in the dorm has at least one friend in the dorm. If no one is regarded as being their own friend, show that in this dorm there are two people who have the an identical number of friends in the dorm.
  - (b) There are four kitchens in the dorm, and each person is randomly assigned to a kitchen . Show that there is a group of 7 people who are assigned to the same kitchen.