Homework/Problems 4

MAT160: Mathematical problems and games.
Spring 2005

02/22/2005. Due on 03/01/2005

Instructions: Solve the following questions.

Let's start with a little reminder about combinatorics. Let n be a natural number. Recall that

$$n! = n(n-1)\dots 2 \cdot 1.$$

By definition we take $0! = 1$.
Let $S = \{s_1, \dots, s_n\}$ a set with n elements. If $k \leq n$, the set of $k-$permutations of S has exactly

$$P(n,k) := \frac{n!}{(n-k)!}$$

elements. Therefore, the number of n-permutations of n different elements is just $n!$. A k-combination of elements in S is a *subset* of S with k-elements. There are exactly

$$\binom{n}{k} := \frac{P(n,k)}{P(k,k)} = \frac{n!}{(n-k)!k!}$$

ways of choosing k elements of S.

Permutations with repetitions

If we have n_1 identical objects of type 1, n_2 identical objects of type 2, ... and finally n_k identical objects of type k, the number of permutations of all these $n_1 + n_2 + \cdots + n_k$ objects is

$$\frac{(n_1 + n_2 + \cdots + n_k)!}{n_1! n_2! \cdots n_k!}.$$

Take a moment to think why this is true.
Example: Having 2 red, 3 green and 5 blue balls, there are $(2+3+5)!/(2!3!5!) = 2520$ different ways to arrange them in a row.

1. In how many ways can you arrange 10 people to sit around a round table. Note that different ways obtained by rotating them around the table will be considered the same.

2. The Hermitian language has only three letters : A, B and C . A word in this language is any sequence of letters (repetitions allowed).

(a) How many four-letter words are there in this language ?

(b) How many words are there which are fewer that ten letters long ?

(c) How many of the nine-letter words start with A ?

3. How many 10 digit numbers have at least 2 digits equal?

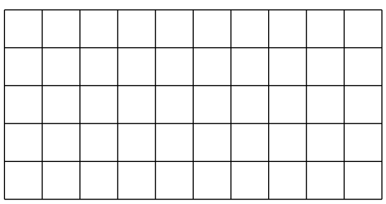
4. For each of the following examples, find the number of different words you can obtain by permuting the letters. (A word doesn't have to make sense.)

"CLOSENESS" "APPLE" "CHEESE" "APPLESAUCE"

5. Simplify the following expressions:

$$\frac{10!}{7!3!} \qquad \frac{n!}{(n-1)!} \qquad \frac{(n+1)!}{n(n+1)}$$

6. How many rectangles are in the following figure? How many squares?



7 In Timbuktu, a social security number is a 5 digit number where each digit is one of $0, 1, 2, 3, 4, 5, 6$. How many different S.S.N. can be in Timbuktu?

How many different S.S. are if 0 cannot be the first digit?

What if in addition they cannot have more than two 1 in one S.S.N.?

8. On a 5×8 board, a rook is placed on the lower left corner of the board. Assume that the rook can only move horizontally to right or vertically upward. In how many different ways, it can reach the upper right corner of the board? (*Hint:* Every possible path corresponds to a word having eight R's, when it is moving a unit rightward, and five U's, when it is moving a unit upward, and every such word determines a path. For example, the word RUURRRURRUURR corresponds to the path determined by moving a unit right, two units up, three units right, a unit up, two units right, two units up and finally two units right to get to the upper right corner.)