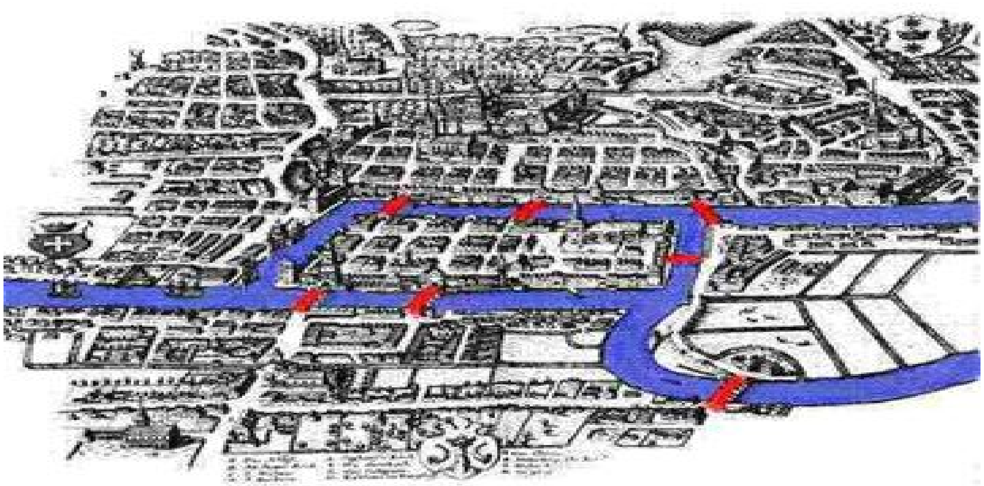
PROBLEM SET 3. MAT160.

Due: Feb 22, 2005.

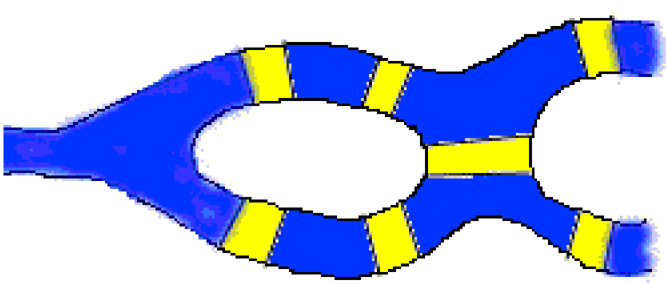
Try to do in class Problems 1,2 and 3.

The Seven Bridges of Konigsberg

In Konigsberg, Germany, a river ran through the city such that in its center was an island, and after passing the island, the river broke into two parts. Seven bridges were built so that the people of the city could get from one part to another.



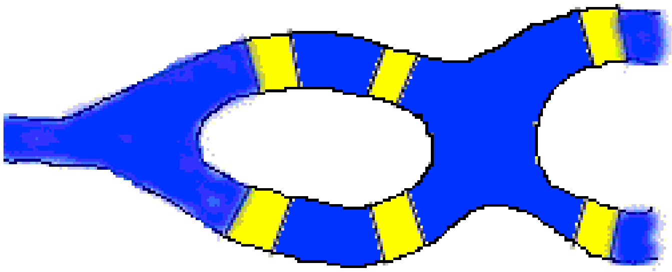
A crude map of the center of Konigsberg might look like this:



The people wondered whether or not one could walk around the city in a way that would involve crossing each bridge exactly once.

1. Try it. Sketch the above map of the city on a sheet of paper and try to 'plan your journey' with a pencil in such a way that you trace over each bridge once and only once and you complete the 'plan' with one continuous pencil stroke.

2. Suppose they had decided to build one fewer bridge in Konigsberg, so that the map looked like this:



3. Does it matter which bridge you take away? What if you add bridges? Come up with some maps on your own, and try to 'plan your journey' for each one.

4. The numbers 1 through 10 are written in a row. Can the signs "+" and "-" be placed between them in such a way that the resulting expression adds up to 0?

5. a) We have a container that contains 9 quarts and another that contains 4 quarts. We fill these containers by immersing them in the river. How can you put exactly exactly 6 quarts of water into the large container?
b) Now suppose that you have a 6 quart container and a 4 quart container.How can we use them to fill one of the containers with 3 quarts of water?

6. Twenty-five men and twenty-five women are seated at a round table. Show that there must be at least one person at the table who is seated between two men.

7. If n is a positive integer such that 2n+1 is a perfect square show that n+1 is the sum of two successive perfect squares.

8. Remove the lower left corner square and the upper right corner square from an ordinary 8-by-8 chessboard. Can the resulting board be cover by 31 dominoes? Assume each domino will cover exactly two adjacent squares of the board.