

## HOMework/PROBLEMS 2

MAT160: Mathematical problems and games.  
Spring 2005

02/08/2005. Due on 02/15/2005

**Instructions:** Solve the following questions.

1. Let  $q$  be a real number other than 1. Use induction on  $n$  to prove that:

$$q^0 + q^1 + q^2 + \dots + q^{n-1} = (q^n - 1)/(q - 1)$$

2. Guess a formula for the sum  $1(2) + 2(3) + 3(4) + \dots + n(n + 1)$  for all  $n$  and prove it by induction.

3. Use induction to prove that a set of  $n$  elements has  $2^n$  subsets.

4. You have 12 coins, one of which is counterfeit. You don't know whether the counterfeit coin is heavier or lighter than the others. Use three weighings on a balance with two pans to find the counterfeit coin.

5. The Checkerboard Problem. Counting squares of sizes one-by-one through eight-by-eight, an ordinary eight-by-eight checkerboard has 204 squares. How can we obtain a formula for the number of squares of all sizes on an  $n$ -by- $n$  checkerboard?

6. How much is  $1 + 3 + 5 + \dots + (2n - 1)$  for all  $n$ ?