

HOMework/PROBLEMS 11

MAT160: Mathematical problems and games. Spring 2005

04/26/2005, due May 3, 2005

Instructions: As usual, think about all problems, and come up with some ideas about how to solve them.

1. A 3x3 table is filled with numbers. It is allowed to increase each number in any 2x2 square by 1. Is it possible, using these operations, to obtain the table shown below starting with a table filled with zeros ?

$$\begin{bmatrix} 4 & 9 & 5 \\ 10 & 18 & 12 \\ 6 & 13 & 7 \end{bmatrix}$$

2. There is a pile of 37 pennies on a table. The first player divides the pile into two smaller piles (not necessarily equal). The second player chooses a pile and divides it into two smaller piles. They continue. Each turn a player chooses one of the piles on the table and divides it into two smaller piles. The player who cannot do this anymore is the loser. Who wins and how?
3. A general needs to take his troops across the river. He spies two boys with a small boat. He commandeers both the boat and the boys. Unfortunately, the boat will only hold two boys or one soldier. Yet he determines a method for getting his troops across. What could be? If there are n soldiers, how many times is the boat going to cross the river in order for everybody to have crossed the river?
4. In a distant land, bigamy is common. There are six people who want to cross a river in this land. This group consists of two men, each with two wives. No man can tolerate any of his wives being in the company of another man unless he is present to chaperon. There is a boat that holds two people to be used for crossing the river. Can they do it? In how many crossings?
5. Same problem as above except the boat will hold three people.
6. All the students in a school are arranged in a rectangular array. The tallest student in each row is identified, and among these students John is the shortest. Then in each column the shortest student is identified, and Mary is the tallest of these. Who is taller: John or Mary ?
7. Prove the Pythagorean theorem. (Hint: you are allowed to use a text-book).
8. On all parallelograms with a given perimeter, which has the greatest area ?