0. Trivial.

1a. Obvious.

1b. Follows from **1c.**

1c. By Problem **0** $(\overline{z})^n = \overline{z^n}$. Hence, $a\overline{z}^n = a\overline{z^n}, \forall z \in \mathbb{C}, \& z \in \mathbb{R}$.
Hence, $\overline{p(z)} = p(\overline{z})$, for all real polynomial p. Hence, $p(z) = 0 \Rightarrow \overline{p(z)} = 0 \Rightarrow p(\overline{z}) = 0$.

2a. Circle of radius 3, centered at $1 + 2i$.

2b. Ellipse with foci at $\pm 4i$, with major axis length 10 and minor axis length 6.

2c. $\{z : |z-1|+|z+1| = 1\} = \emptyset$.Since, $2 = |1-(-1)| \leq |z-1|+|z+1|, \forall z \in \mathbb{C}$ (by triangle inequality).

2d. The perpendicular bisector of the straight line segment joining 1 and i.

In the solutions of problem **3** the regions will always exclude the boundary, unless otherwise mentioned explicitly.

3a. Area inside the circle of radius 1, centered at i.

3b. Area outside the circle of radius 3, centered at $1 + 2i$.

3c. Area between the circles of radii 3 and 1 centered at 1, including the inner boundary.

3d. Area above the $x(real) - axis$.

3e. The wedge defined by positive real axis and the part of the line $x = y$ in the first quadrant. It includes the boundaries, but the origin.

3f. The area under the straight line (not just the segment) joining 1 and i.

3g. Area between the lines $x = y$ and $x = -y$, which incldes the x-axis but the origin. If, $z = x + iy$, then,

$$Re(z^2) = Re((x+iy)^2) = Re(x^2 - y^2 + 2ixy) = x^2 - y^2$$

. Therefore, $Re(z^2) > 0 \Rightarrow x^2 > y^2 \Rightarrow |x| > |y|$.

4. For $n = 2$.

$(\cos(2\theta) + i\ sin(2\theta)) = (cos\theta + i\ sin\theta)^2$
$=\cos^2\theta - sin^2\theta + i\ sin\theta\ cos\theta$
$=2\cos^2\theta - 1 + i\ sin\theta\ cos\theta$(by Pythagorus)

Now, equating the real parts of both the equation, $cos2\theta = 2cos^2\theta - 1$.

For $n = 3$, similar exercise will give you, $cos3\theta = 4cos^3\theta - 3cos\theta$.

5. Observe that, $(1+i) = \sqrt{2}(cos\frac{\pi}{4} + i\ sin\frac{\pi}{4})$. Therefore, by de Moivre,

$(1+\mathrm{i})^{2n} = 2^n(cos\frac{n\pi}{2} + i\ sin\frac{n\pi}{2})$

Now expand the left side of the equation to find that the big expression in the question is actually the real part of the left side. And then equate the real parts of the equation to get the result.